元 智 大 學

工業工程與管理研究所

碩士論文

**Application of Forecasting Methods to Slow Moving Demand Items: A Case Study**

**Student: Fabio Esteban Sierra Gayon**

**Advisor: Dr. Chi-Yang Tsai**

**中華民國 110年 07 月**

**Application of Forecasting Methods to Slow Moving Demand Items: A Case Study**

Student: Fabio Esteban Sierra Gayon

Advisor: Dr. Chi-Yang Tsai

元 智 大 學

工業工程與管理研究所

碩士論文

A Thesis Submitted to Institute of Industrial Engineering and Management

Yuan Ze University

In partial Fulfillment of the Requirements for the Degree of Master in the Department of Industrial Engineering and Management

2021

Zhongli, Taiwan, Republic of China

**TABLE OF CONTENTS**

[TABLE OF FIGURES 6](#_Toc97482670)

[TABLE OF TABLES 8](#_Toc97482671)

[CHAPTER 1 INTRODUCTION 9](#_Toc97482672)

[1.1. Research background 9](#_Toc97482673)

[1.2. Research motivation 10](#_Toc97482674)

[1.3. Research objectives 12](#_Toc97482675)

[CHAPTER 2 LITERATURE REVIEW 13](#_Toc97482676)

[2.1. Background about forecasting 13](#_Toc97482677)

[2.2. Categorization of demand patterns 14](#_Toc97482678)

[2.3. The Croston Method 17](#_Toc97482679)

[2.4. The Croston Method Modifications 19](#_Toc97482680)

[2.5. The Aggregate-Disaggregate Intermittent Demand Approach (ADIDA) and the Multiple Aggregation Prediction Algorithm (MAPA) 21](#_Toc97482681)

[CHAPTER 3 CASE STUDY 26](#_Toc97482682)

[3.1. Case Introduction 26](#_Toc97482683)

[3.3. Data Description 28](#_Toc97482684)

[3.4. Problem Formulation 29](#_Toc97482685)

[3.5 Application of forecasting methods for slow moving items 33](#_Toc97482686)

[CHAPTER 4 RESULTS AND DISCUSSION 37](#_Toc97482687)

[4.1 Croston’s and Exponential Smoothing Methods 37](#_Toc97482688)

[4.1.1 Erratic Items 37](#_Toc97482689)

[4.1.2. Intermittent items 48](#_Toc97482690)

[4.1.3 Lumpy demand items 57](#_Toc97482691)

[4.2. Aggregate-Disaggregate Intermittent Demand Approach (ADIDA) and Multiple Aggregation Prediction Algorithm (MAPA) 65](#_Toc97482692)

[4.2.1. Erratic items 65](#_Toc97482693)

[4.2.2. Intermittent items 69](#_Toc97482694)

[4.2.3. Lumpy demand 73](#_Toc97482695)

[4.3. Final comparison between the aforementioned methods 76](#_Toc97482696)

[4.3.1 Erratic demand items 76](#_Toc97482697)

[4.3.2. Intermittent demand items 77](#_Toc97482698)

[4.3.3. Lumpy demand items 78](#_Toc97482699)

[REFERENCES 80](#_Toc97482700)

# TABLE OF FIGURES

[Figure 1 Demand pattern classification (Williams, 1984) 15](#_Toc97482701)

[Figure 2 William’s categorization scheme (Syntetos et al., 2005) 15](#_Toc97482702)

[Figure 3 Demand pattern classification (Syntetos et al. 2005) 16](#_Toc97482703)

[Figure 4 Demand patterns (Costantino et al, 2018) 17](#_Toc97482704)

[Figure 5 Croston’s method algorithm (Şahin et al., 2013 and Xu et al., 2012) 18](#_Toc97482705)

[Figure 6 ADIDA forecasting framework (Nikolopoulos et al., 2011) 22](#_Toc97482706)

[Figure 7 Managerial and Systemic Viewpoint of the ADIDA framework (Spithourakis et al., 2014) 23](#_Toc97482707)

[Figure 8 Aggregation levels of a times series (Kourentzes et al., 2014) 24](#_Toc97482708)

[Figure 9 Forecasting and combination of a times series (Kourentzes et al, 2014) 25](#_Toc97482709)

[Figure 10 Agrosavia’s warehouses scheme 27](#_Toc97482710)

[Figure 11 Demand pattern of the items in Agrosavia 30](#_Toc97482711)

[Figure 12 Number of items by type of demand in Agrosavia 31](#_Toc97482712)

[Figure 13 BIP001271 demand pattern (erratic demand) 31](#_Toc97482713)

[Figure 14 BIP008013 demand pattern (intermittent demand) 32](#_Toc97482714)

[Figure 15 BIP005887 demand pattern (Lumpy demand) 32](#_Toc97482715)

[Figure 16 Scheme for evaluating the forecasting process of Agrosavia’s items 36](#_Toc97482716)

[Figure 17 Item BIP001271 demand behavior 38](#_Toc97482717)

[Figure 18 Croston's and Expo. Smooth. forecasts of item BIP001271 with α = 0.1 39](#_Toc97482718)

[Figure 19 Croston's and Expo. Smooth. forecasts of item BIP001271 with α = 0.15 41](#_Toc97482719)

[Figure 20 Croston's and Expo. Smooth. forecasts of item BIP001271 with α = 0.5 43](#_Toc97482720)

[Figure 21 Croston's and Exp. Smooth. forecasts of item BIP001271 with α = “opt.” 44](#_Toc97482721)

[Figure 22 Item BIP001271 demand behavior 48](#_Toc97482722)

[Figure 23 Croston's and Exp. Smooth. forecasts of item BIP008013, alpha = 0.1 49](#_Toc97482723)

[Figure 24 Croston's and Exponential Smoothing forecasts of item BIP008013, alpha = 0.15 51](#_Toc97482724)

[Figure 25 Croston's and Exp. Smooth. forecasts of item BIP008013, alpha = 0.5 52](#_Toc97482725)

[Figure 26 Croston's and Exp. Smooth. forecasts of item BIP008013, α = “opt.” 54](#_Toc97482726)

[Figure 27 Item BIP005887 demand behavior 57](#_Toc97482727)

[Figure 28 Croston's and Exp. Smooth. forecasts of item BIP005887, α = 0.1 58](#_Toc97482728)

[Figure 29 Croston's and Exp. Smooth. forecasts of item BIP005887, α = 0.15 59](#_Toc97482729)

[Figure 30 Croston's and Exp. Smooth. forecasts of item BIP005887, α = 0.5 61](#_Toc97482730)

[Figure 31 Croston's and Exp. Smooth. forecasts of item BIP005887, α = “opt.” 62](#_Toc97482731)

[Figure 32 Aggregation levels for Item BIP001271 66](#_Toc97482732)

[Figure 33 Aggregation-Disaggregation forecasting methods for item BIP001271 68](#_Toc97482733)

[Figure 34 Aggregation levels for Item BIP008013 71](#_Toc97482734)

[Figure 35 Aggregation-Disaggregation forecasting methods for item BIP008013 72](#_Toc97482735)

[Figure 36 Aggregation levels for Item BIP005887 74](#_Toc97482736)

[Figure 37 Aggregation-Disaggregation forecasting methods for item BIP005887 75](#_Toc97482737)

# TABLE OF TABLES

[Table 1 Monthly demand of Agrosavia’s 30](#_Toc97482738)

[Table 2 Smoothing and forecast of item BIP001271 40](#_Toc97482739)

[Table 3 Errors of item BIP001271 forecast with α = 0.1 41](#_Toc97482740)

[Table 4 Errors of item BIP001271 forecast with α = 0.15 43](#_Toc97482741)

[Table 5 Errors of item BIP001271 forecast with α = 0.5 45](#_Toc97482742)

[Table 6 Errors of item BIP001271 forecast with α = “optimal” 47](#_Toc97482743)

[Table 7 Smoothed inter-demand interval of item BIP001271 (erratic demand) 48](#_Toc97482744)

[Table 8 Errors of item BIP008013 forecast with α = 0.1 51](#_Toc97482745)

[Table 9 Errors of item BIP008013 forecast with α = 0.15 53](#_Toc97482746)

[Table 10 Errors of item BIP008013 forecast with α = 0.5 54](#_Toc97482747)

[Table 11 Errors of item BIP008013 forecast with α = “optimal” 56](#_Toc97482748)

[Table 12 Smoothed inter-demand interval of item BIP008013 (intermittent demand) 57](#_Toc97482749)

[Table 13 Errors of item BIP005887 forecast with α = 0.1 60](#_Toc97482750)

[Table 14 Errors of item BIP005887 forecast with α = 0.15 61](#_Toc97482751)

[Table 15 Errors of item BIP005887 forecast with α = 0.5 62](#_Toc97482752)

[Table 16 Errors of item BIP005887 forecast with α = “optimal” 64](#_Toc97482753)

[Table 17 Smoothed inter-demand interval of item BIP005887 (intermittent demand) 65](#_Toc97482754)

[Table 18 Selection of forecasting method for ADIDA and MAPA - BIP001271 68](#_Toc97482755)

[Table 19 Aggregation – Disaggregation smoothing and forecast of item BIP001271 70](#_Toc97482756)

[Table 20 Aggregation errors for item BIP001271 70](#_Toc97482757)

[Table 21 Selection of forecasting method for ADIDA and MAPA - BIP008013 71](#_Toc97482758)

[Table 22 Table 20 Aggregation errors for item BIP008013 74](#_Toc97482759)

[Table 23 Selection of forecasting method for ADIDA and MAPA - BIP005887 74](#_Toc97482760)

[Table 24 Table 22 Table 20 Aggregation errors for item BIP005887 77](#_Toc97482761)

# CHAPTER 1 INTRODUCTION

## Research background

Inventory management has been a long-standing topic in Operations Research. Silver (1981) mentions 8 major problems that are evident in this area of study, from deterministic or probabilistic demand, through the nature of the supply process, to the structure of inventory costs or the useful life of the stock, to mention some of them. However, most of the items that are analyzed in these problems present a constant demand, i.e. that within their variables there is demand in all the periods being analyzed. This is how the study of slow moving items enters the scene.

Firstly, slow moving items have a singular behavior as they have a slow demand, i.e. a demand that fluctuates from low to high during the periods in which it exists, but which has a large number of zeros in the periods under analysis. This can also be defined according to the analysis of the Coefficient of Variation presented by its demand, as well as the Average Inter-Demand Interval, which allows the classification of the demands of these items into smooth, intermittent, erratic and lumpy (Syntetos et al., 2005). Furthermore, this kind of items are very representative in the storages of companies, because they can represent, approximately, up to the 60% of references in a company (Williams, 1984 and Johnston et al., 2003)

With this in mind, the most used forecasting methods to know future demand have been very varied, being exponential smoothing one of them, thanks to the ease with which it can be calculated, as well as the certainty of its results (Gardner, 1985). However, in the case of slow-moving items with intermittent demands, it is not a successful method, because it presents irregularities in the performance of the results (Silver et al., 1998), as well as thanks to the appearance of many zeros during the periods, the forecast will tend towards this value (Wallstrom & Segerstedt, 2010).

For this reason, several forecasting models have been developed, based on the classification of demands. Thus, one of the most important and the one that gave rise to the most accurate results was the one proposed by Croston (1972). This method separates the probability in which the demand occurs and the size of this when it exists, so that it can avoid the bias that appears when using the exponential smoothing. Similarly, to reduce this bias, another variable is introduced, such as the estimated inter-arrival time each demand occurs.

However, some modifications have emerged from this method that have generated more precise results with fewer biases, such as the use of two previous periods, instead of one as in the original (Vinh, 2005). On the other hand, Croston method generated bias, mainly in how the smoothing constant was used, so that in Syntetos & Boylan (2005) the estimator of mean demand involves this constant to update the inter-demand intervals. This is then multiplied by the average demand of the period, based on the Croston method and, thus, correcting this bias. This can be found too in the correction done by Shale et al. (2006), where

On the other hand, there are also other methods that, like those mentioned above, generate results to predict intermittent demands. Such is the case of ADIDA (Aggregate-disaggregate Intermittent Demand Approach), in which the original time series are grouped in larger time units, i.e., monthly data are grouped into bimonthly or quarterly data, this new series is forecast and then it is disaggregated into the same original time unit (Nikolopoulos et al., 2011).

Based on these intermittent demand forecasting methods, there are several fields in which they can be used, not only in the study of the behavior of slow-moving items, but also in the occurrence of natural disasters or data analysis (Nikolopoulos, 2021).

## Research motivation

Most of the studies carried out to the Operations Research, specifically related to the management and control of the inventory, are applied to problems in which the companies are in charge of the production of spare parts or in the production of some article, because most of them respond to the reduction of production costs or the maximization of profits. However, many of them neglect the application of the models developed in this areas to other fields of action, such as companies that directly maintain inventories, but not for the production of articles. The case for which this research is to be carried out is of the Colombian company Agrosavia, which is in charge of developing and executing research, technology, and transfer of technological innovation processes to the agricultural sector and it would be a valuable opportunity to generate some recommendations so that its operation and inventory planning will be more optimal.

Similarly, it is important to highlight that forecasting is one of the most important tasks, within the planning of the operations of a company. Thanks to it, it is possible to anticipate what the future holds, due to the historical behavior of the data to be analyzed. In the case of the company to which this study will be the subject, the demand for basic operating supplies will be the main product to be analyzed.

Within the forecasting methods that exist today, all of them have been applied to environments where the demand has been constant, i.e., in each period there is a constant consumption of products, whose variation in the size of the demand is minimal. However, there are cases in which there are items, whose demand is sporadic or slow moving demand items. That is the case of the company to be studied, most of its products present this type of demand, which leads them to carry out purchase orders in a disorganized way, to have high inventories in periods where it is not necessary to have them, leading to increase their holding companies. costs.

In this way, the study proposes to evaluate the type of demand for basic operating products that Agrosavia has, through the Coefficient of Variation and the Average Inter-Demand Interval (ADI), so that certain forecasting models can be applied to items with slow moving demand, mainly those that are based on the Croston method, who was the pioneer in studying this phenomenon, and aggregation approaches. Besides, the methods based on Croston and aggregation generate more accurate forecasts than traditional methods, such as Simple Exponential Smoothing or methods related to Moving Average.

## Research objectives

The main objective of this research is to analyze the behavior of the demand for the supplies of the Agrosavia company inventory system, according to the forecast models applied to the most well-known slow moving demand problems in order to recommend to this company the application of the one that presents the best results. In this way, the specific objectives will be seen down below:

1. To classify the demands of the supplies in the Agrosavia inventory system.
2. To apply forecasting methods specifically designed for slow-moving demand patterns.
3. To measure the precision of these models, by comparing the errors they demonstrate
4. To recommend to the company the most practical method and to present the most accurate predictions.

# CHAPTER 2 LITERATURE REVIEW

## 2.1. Background about forecasting

The ability to forecast can be defined as the prediction of an event that may occur in the future. Usually, this tool can be used in various economic sectors, such as industry, businesses, government, environmental sciences, finance, among others, through the use of time series, or chronological observations of a specific variable, which can range from the viscosity of a product, until the demand for some item (Montgomery et al., 2016). In the same way, it is one of the most important tasks associated with Operational Research and Supply Chain Management, thanks to the fact that with forecasts, more accurate decisions can be made (Nikolopoulos, 2021).

Within Supply Chain Management, many of the actors that are within it seek to optimize their inventories, which is manifested in the reduction of their costs, i.e., improve the accuracy of forecasts and reduce their inventory levels, in addition to improve business competitiveness (Tliche et al., 2020). The lack of forecasting processes or their poverty can lead to situations of stock out or overstock, which would reduce profits and customer satisfaction (Steenbergen & Mers, 2020). For this, in some cases it is necessary that the method to be applied has to be simple and easy to apply and interpret (Stevenson, 2009 and Tliche et al., 2020). Some of these are so simple, such as Naive, where the predicted period is equal to the immediately preceding period (Stevenson, 2009; Ryu & Sanchez, 2003; Spithourakis et al., 2014), but with the high probability that its precision is not very adequate. On the other hand, there are more complex and widely used methods, in which the mean is used as a mechanism to smooth the variations in the data. Among these, there is the Simple Moving Average (SMA), the Weighted Moving Average (WMA), using different weights as the values are more recent, or the Simple Exponential Smoothing (SES), where each forecast is based on the previous one, adding a percentage of error to the difference between these two values (Croston, 1972; Gardner, 1985; Syntetos & Boylan, 2005; Billah et al., 2006; Stevenson, 2009; Wallstrom & Segerstedt, 2010).

However, in most cases where these methods are applied, the time series are constant or show a definite pattern, mainly in the forecast of demand. There are other scenarios where this indicator presents irregularities, so that the previously proposed methods are not enough to generate correct forecasts (Dunsmuir & Snyder, 1989; Johnston et al., 2003; Uzunoglu & Tamer, 2011; Santa Cruz & Corrêa, 2017)

## 2.2. Categorization of demand patterns

Within the issues related to inventory management, it is important to note that the classification of items that a system has facilitates the application of techniques or models to process them. The ABC classification, one of the best known, distributes in each of these letters the monetary weights and the quantity that an item represents in the inventory. However, the classification of items through the behavior of their demands shows those that have fast moving demand, which common forecasting methods would be applied (Syntetos & Boylan, 2005), and slow moving demand.

To categorize demand, Williams (1984) proposes a method in which he performs a partition of the demand variance during the lead time (DDLT), into smaller parts, that is, in the variance of the number of orders arriving, the variance of the size of these orders and the variance of the lead times, giving rise to the following equation:

(1)

Where *λ* is the mean of the number of demands, occurring at a random stream, is the mean of the lead time and *Cx*, is the coefficient of variation of the distribution of order size. Hence, the two products obtained can be explained separately: represents the mean number of lead times between demands, while represents the intermittence in demand. Thus, Figure 1 shows how the interaction of these two variables is observed, establishing 4 types of demands: type A or low sporadicity, B or slow-moving, C or frequent demand with large variable sizes, and D or with high sporadicity.

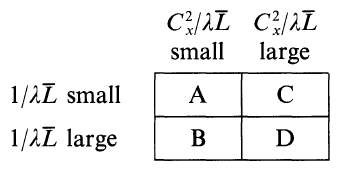


Figure Demand pattern classification (Williams, 1984)

To determine the maximum limits of lead times and intermittency at which an item could be in certain categories, Williams (1984) proposed Figure 2, which additionally shows a division of the items with type D demand into sporadic demand, D1, and highly sporadic demand, D2. (Syntetos et al, 2005).

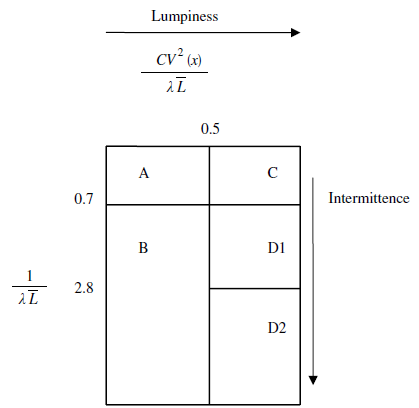


Figure William’s categorization scheme (Syntetos et al., 2005)

Later, the classification of demands would be modified a little. Thanks to the comparison of the EWMA, Croston and SBA methods (which will be explained later), mainly their MSEs, to establish which of them could better predict slow moving items, Syntetos et al. (2005) propose new cut-off points to the model proposed by Williams, which would be defined by calculating the square of the Coefficient of Variation and the Average Inter-Demand Interval (p). In the same way, it is evident in Figure 3 that the intermittent Williams demand patterns are consolidated into a single category (Boylan et al, 2008).

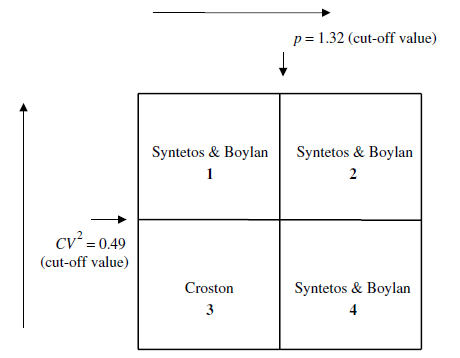


Figure Demand pattern classification (Syntetos et al. 2005)

As can be seen, the categories are practically divided in the same way as Williams. However, the names given to each of them are as follows:

1. Erratic, but not very intermittent.
2. Lumpy
3. Smooth
4. Intermittent, but not very erratic.

It is important to define each of them. An erratic demand means that the size of it is highly variable. On the other hand, intermittent demand has the characteristic that it occurs infrequently. As for lumpy, it could be said that it is the fusion of intermittent and erratic, that is, an intermittent demand that, when it occurs, is highly variable (Boylan et al., 2008). Meanwhile, smooth demand is the most constant of them. Observing it graphically, the classifications of the demand patterns can be similar to the following examples, where it can be seen the limit points of the squared Coefficient of Variation and the Average Inter-Demand Interval (ADI):

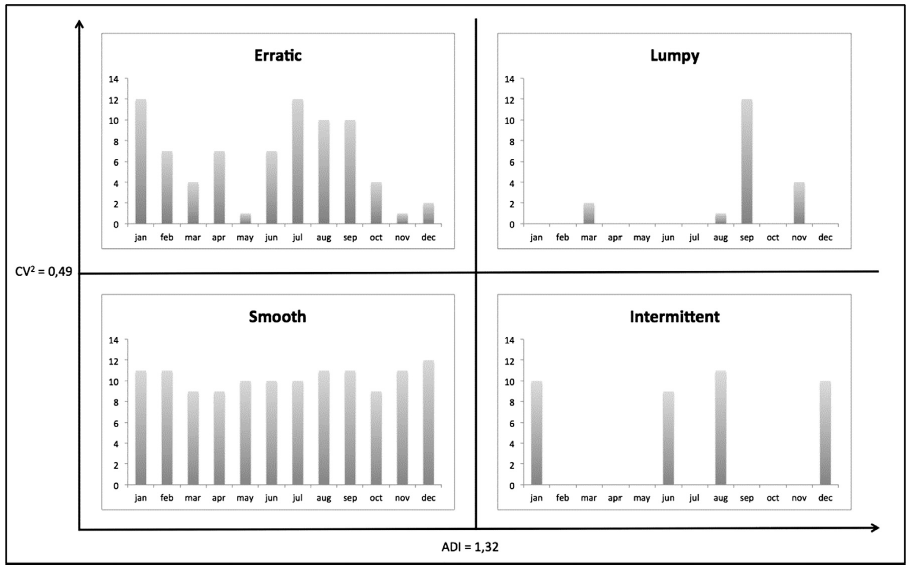


Figure Demand patterns (Costantino et al, 2018)

## 2.3. The Croston Method

Commonly, to forecast the demand for some product, one of the most used methods is Simple Exponential Smoothing. This tool is very popular, thanks to the ease that it can be executed and the generation of precise results (Gardner, 1981). However, when focusing on more specific problems, such as the forecast of slow moving items and the appearance of large amounts of zeros in the time series, the results may tend towards this value, losing all precision (Wallstrom & Segerstedt, 2010).

Croston (1972), proposes a two-part model that modifies the SMS to give more accurate results. These two parts are the size of the demand and the number of periods that occur between demands, i.e., the inter-demand interval. With these two variables, he establishes the following algorithm:

Start

Next t

End

No

Yes

Yes

No

Figure Croston’s method algorithm (Şahin et al., 2013 and Xu et al., 2012)

Where:

* : demand in period *t*
* : forecast of the next non-negative demand beyond the period *t*
* : Forecast of the average demand per period
* : forecast of demand interval
* : interval since the last non-negative demand
* : smoothing constant
* : smoothing parameter for inter-demand intervals

In this way, it can be seen in figure 5 that this method is only updated if there is demand in the period in which it is going to be forecast, i.e., if the demand in the period is equal to zero (), the forecast () will be the same value that was predicted in the previous period, similar to the forecast of the demand interval (). In the end, the ratio between () and () will be the average estimate of demand for the next period.

This method has been used in several cases where the problem of slow moving items exists. Xu et al. (2012) compiles several of the studies in which this method contributed to the study of the behavior of these items. Among them is the improvement in the results, compared with traditional methods such as EWMA (Willemain et al, 1994), mainly with ADI's higher than 1.25 (Johnston & Boylan, 1996) or using less than 0.15, because as this constant increases, there is an increase in the forecasting bias, which gave rise to the first modification to the Croston method or the SBA method, in honor of its authors (Syntetos & Boylan, 2005).

## 2.4. The Croston Method Modifications

One of the most interesting modifications that the Croston Method has undergone, explains that, instead of taking only the previous period with non-negative demand to make the forecast by demand period (), the last two should be taken . This would specify the effects of the forecast in general, besides that thanks to the irregular demand, the past trends would not affect the final result. This, when measured and compared with the Mean Square Errors (MSE) of the Croston method, showed greater precision (Vinh, 2005). This modification is calculated as follows:

(2)

This was applied too to the forecast of the demand interval (), resulting in this formula:

(3)

For the average demand per period , the ratio is calculated in the same way as in the Croston mode.

After having made the corresponding comparisons and evaluating the existing forecasting methods, Syntetos and Boylan (2005) find that the Croston method presents bias, mainly when calculating the average demand in the period (). In this way, they propose that the ratio between and be multiplied by the smoothing constant for inter-demand intervals divided by two and subtracted from 1, like this:

(4)

To verify that the results of the SBA model were solid, they were compared with the Croston method, SES and Simple Moving Average and, thanks to the evaluation of the performance of the Mean Signed Errors (thanks to its ease of calculation and its easy interpretation) and the Geometric Root Mean Square Error (since it immediately compares the level of precision of all the methods), it was unanimous that this new method corrected the bias and presented more accurate forecasts (Syntetos & Boylan, 2005).

However, the fact that these modifications exist does not prevent the Croston Method from being used. In some cases, it presents better performance when applied to items with smooth demands (Syntetos et al., 2005 and Kaya et al., 2020) or to items with intermittent demands (Kaya et al., 2020). Even other modifications to the Croston method were based on the correction of the smoothing factor proposed by Syntetos and Boylan, but extending the use to demands with Erlang and Gamma type distributions (Shale et al., 2006). In this last case, the correction factor replaces the one proposed in the SBA, hence the forecast of the Average Demand will be denoted by:

(5)

However, the study of forecasting models for slow moving items does not apply only to these mentioned ones. Leven and Segerestedt (2004) used the same Croston method, but this was updated as long as there was positive demand, which would update the predicted periodic demand, using the formula proposed by Croston. However, this method was not successful, as it continued to show bias (Boylan & Syntetos, 2007 and Segerstedt & Leven, 2019). On the other hand, another related method is the TSB, granted by the initials of its creators (Teunter et al, 2011), in which the demand is modeled and updated probabilistically, instead of the demand interval, which would allow the demand Each period will be updated, not only when a period has positive demand, so that the obsolescence of the items can be viewed more easily and how this affects their demand.

## 2.5. The Aggregate-Disaggregate Intermittent Demand Approach (ADIDA) and the Multiple Aggregation Prediction Algorithm (MAPA)

One of the least explored factors in the analysis of time series of items with slow moving demand is the possibility that these are subject to a temporal aggregation. This is understood as an activity in which a low-frequency time series, such as bimonthly or biannual, originates from a high-frequency one, such as monthly or weekly. The application of forecasts to series with high aggregation, usually, is more precise, than with its counterparts with low aggregation. If this is applied to a series in which the majority of periods are composed of zeros, the periods with demand would be closer, so the intermittency would be reduced and it would be easier to forecast with simpler methods. The ADIDA method (Aggregate-Disaggregate Intermittent Demand Approach) (Nikolopoulos et al., 2011) has within its philosophy this concept explained above. Additionally, this method tends to perform a non-overlapping aggregation.

The way in which the authors developed this method was exemplifying a monthly series with high intermittency. Later, it was added to a new time series on a quarterly basis, showing a much more uniform demand, but with inherent instability. In this way, a forecasting method is applied that allows the next period of the aggregated series to be extrapolated. Once this is done, the quarterly series is disaggregated into the original time units, for the following months, according to the same weights, i.e., if they are the next 4 months, 1/4. (Nikolopoulos et al., 2011)

The graphic form of this description can be seen in figure 6:

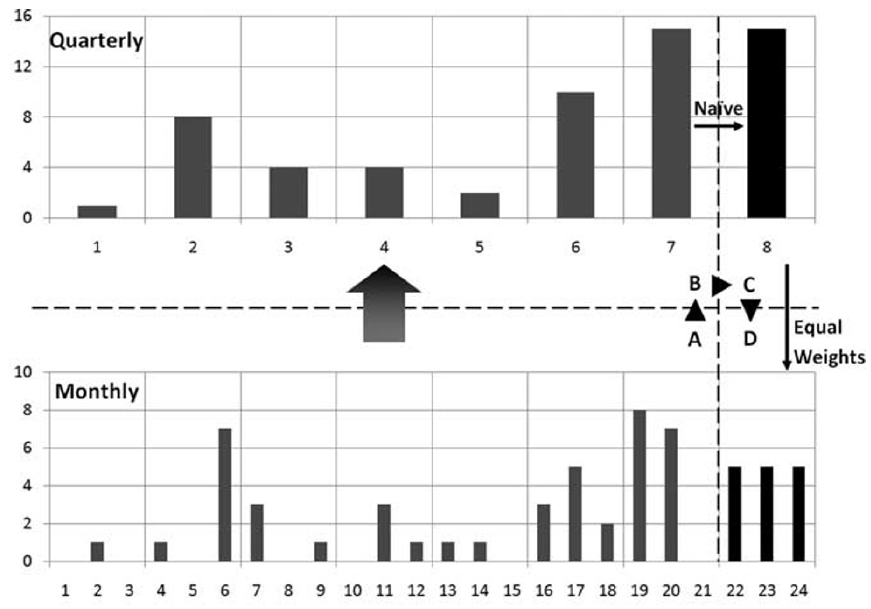


Figure ADIDA forecasting framework (Nikolopoulos et al., 2011)

Where (Spithourakis et al., 2014):

* (A) Obtaining the original data on its lowest timescale.
* (B) Performing the aggregation at a lower level. This step has nomenclature A for aggregation.
* (C) Application of forecasting method to extrapolate. This step has nomenclature F for forecasting.
* (D) Disaggregation of the forecast on the original time scale. This step has nomenclature D for disaggregation

Given that this method facilitates the flexibility of the use of other tools during the forecasting part, as well as its application to intuitive thinking, ADIDA can also be viewed from a systematic point of view (Spithourakis et al., 2014), where the aggregation steps (A) and disaggregation (D), have equivalences with the application of the Simple Moving Average (SMA) and the Weighted Moving Average (WMA), respectively. This can be seen in Figure 7.

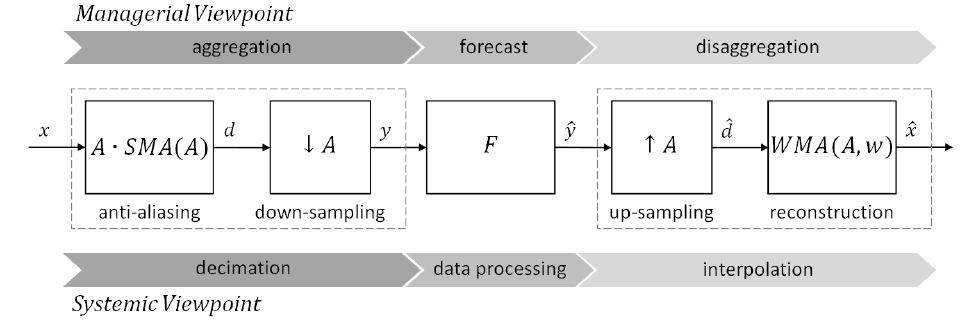


Figure Managerial and Systemic Viewpoint of the ADIDA framework (Spithourakis et al., 2014)

Where:

* : initial time series
* : averaged time series (i.e. series after SMA filtering)
* : aggregate time series
* : forecast model for aggregate series
* : downsampled aggregate forecast model
* : forecast model for initial series

The systematization of the ADIDA method facilitates the standardization of each of its steps, in a mathematical way, so that the data is reduced through a decimation process, preceded by the SMA, which becomes an anti-aliasing filter. By applying an extrapolation method to forecast (a method that can be used freely), data processing is performed. At the end, the aggregate series is interpolated, disaggregated by means of the WMA as a reconstruction filter (Spithourakis et al., 2014)

However, one of the main drawbacks of ADIDA's approach is to know what is the optimal level of aggregation to which the original series should adhere. Nikolopoulos et al. (2011) proposed that the best way was to use the lead time plus a review period, which would help improve decision-making for inventory control. In this way, to review and obtain a better optimization in the search for the level of aggregation, Kourentzes et al. (2014) proposed an algorithm called MAPA or Multiple Aggregation Prediction Algorithm, even combining it with slow-moving items forecasting methods (Petropoulos & Kourentzes, 2014). This method takes the philosophy of ADIDA, on aggregation and disaggregation, but instead of taking a single level, it considers time series subjected to several levels of aggregation simultaneously, obtaining the benefits that each of them can bring to the final forecast, and then combine them. all in a final forecast.

In principle, this algorithm has three stages: aggregation, forecasting and combination. In the first of them, taking the arithmetic mean as the aggregation operator, a time series is added at different levels of aggregation, that is, if it is monthly, it can be added to up to 12, so that this would be a level in which the data are grouped by year (which has 12 months), as seen in Figure 8 and the power indicators, showing where the seasonality or trends are visible. Having all the aggregation possibilities, these must be predicted.

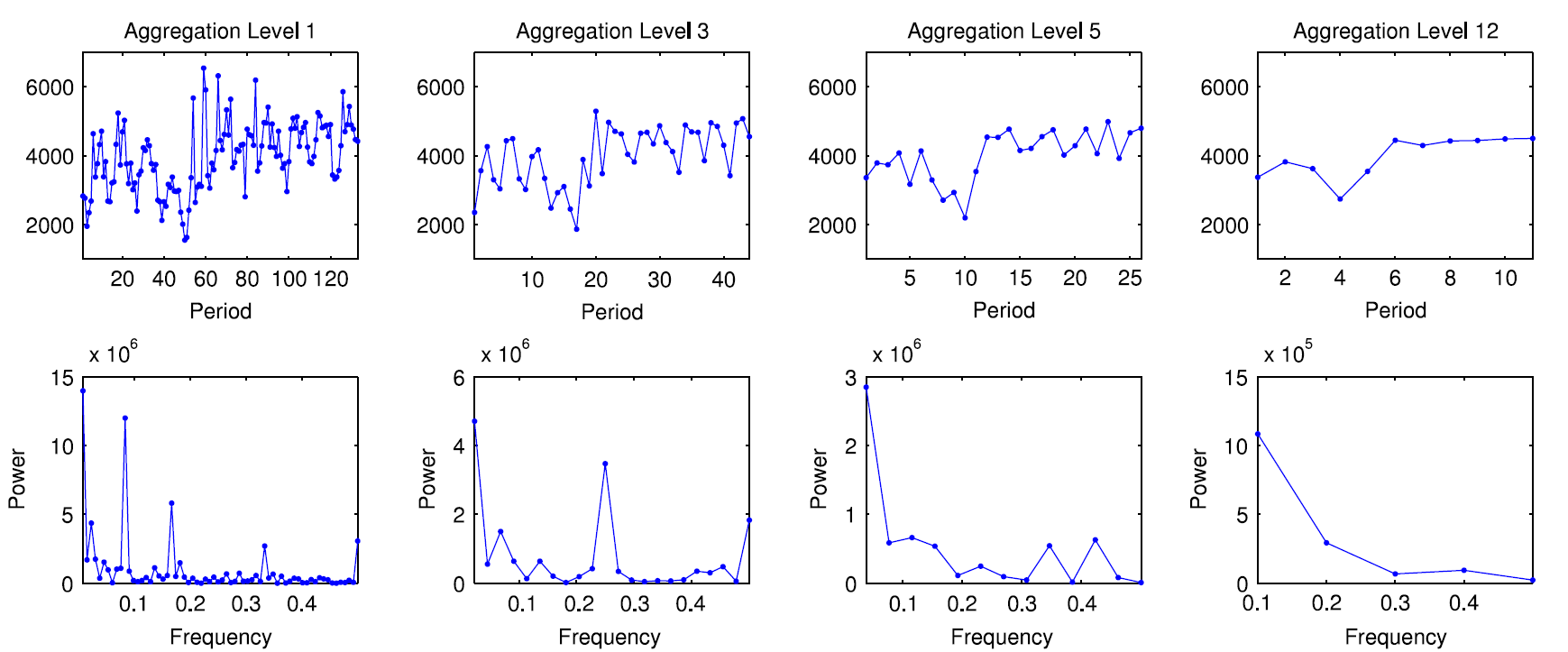


Figure Aggregation levels of a times series (Kourentzes et al., 2014)

Due to its flexibility and its wide use, the models based on the Simple Exponential Smoothing are applied to these time series, depending on the type of component that it presents, that is, if it has seasonality or trend. As it is seen in Figure 9, these components are transformed into others with additive characteristics, to later obtain the final components that will be combined in the last step. Finally, to combine them successfully, each component must be mixed independently of the others, without the need to adjust their level thanks to the use of the mean as an operator, through the unweighted mean and the median.

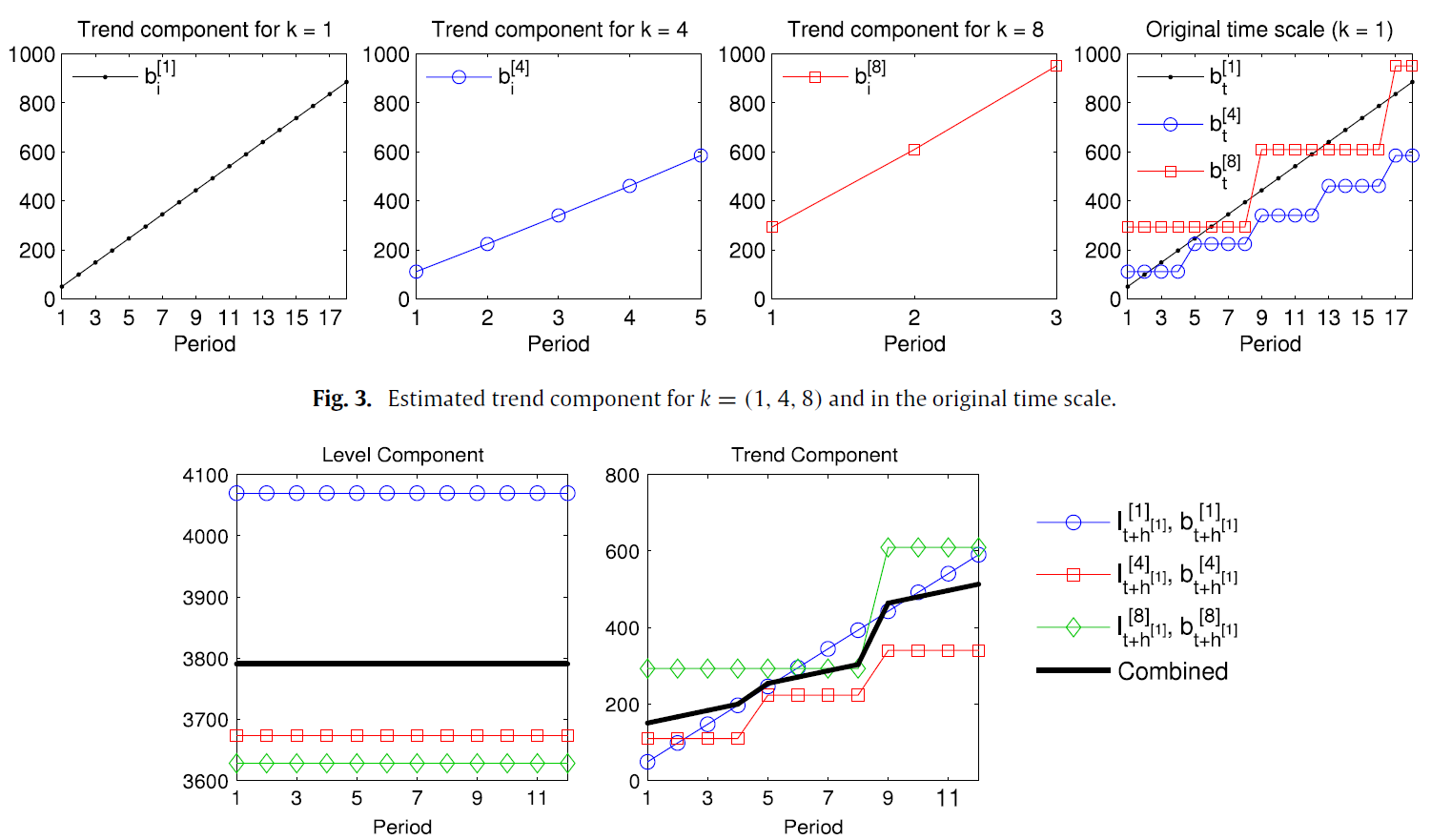


Figure Forecasting and combination of a times series (Kourentzes et al, 2014)

# CHAPTER 3 CASE STUDY

## 3.1. Case Introduction

As mentioned above, the company under study is the Agrosavia agricultural research center, which has several laboratories where the necessary experimentation processes are carried out. In addition, Agrosavia has several research centers throughout the country. However, where this research will focus is the main headquarters (Tibaitata), since it is the largest and where most of the research laboratories are located.

In relation to the subject of inventory, Agrosavia uses a great variety of materials, because each laboratory needs very specific chemical elements, which will only be needed at essential moments and will not be analyzed in this research. However, the materials that this entity calls "basic equipment" are those that the laboratories, as well as administrative and other areas, use to develop their basic operations, i.e., the ones that are ordered and consumed more frequently. It should be noted that some of them are also chemical elements, but these are materials that have a more common demand, because, although they are specific to each project, they are only ordered when starting or developing a project and, usually, these are different, which will make them last in the laboratory warehouse for a long time.

This company has a scheme of warehouses, divided into 3 echelons:

* A main warehouse, which conglomerates all the necessary supplies for the operation and distributes to the administrative units of the company
* An intermediate warehouse called "Cathedral", which receives supplies from the main warehouse and distributes exclusively to research laboratories.
* The laboratory storage, which receive the submissions from Cathedral and, to a lesser extent, from the main warehouse.

Principal warehouse

Cathedral

Figure Agrosavia’s warehouses scheme

In this way, the company has an ERP that allows it to manage internal processes, including requests for items that each laboratory needs. However, Agrosavia does not sell any of the items requested, because these are used for research. Besides, there is no control of the demand for items that are requested from the main warehouse. For this reason, there have been cases of products that remain in the main warehouse or in the Cathedral for a long time, generating cost overruns. For the development of this research, due to the lack of defined demand, the outputs of articles from the warehouses will be used as the demand for the corresponding item, because when an article leaves the warehouse, it is because it has been used.

**3.2. Problem Description**

The main inventory problem that this company has is that most of the items have very few outlets, that is, the demand for them is very slow. This indicates that the company does not adequately forecast the consumption of these supplies, in addition to the fact that conventional methods do not generate accurate results, due to the large number of periods that present demands equal to zero. Similarly, the company does not present a classification of these items according to its demand, so it would be inaccurate to apply a standard forecasting method to items with irregular behavior.

## 3.3. Data Description

The data that was used to do this research was provided directly from Agrosavia, from December 31, 2017 to November 2020. These two dates are taken, because the first marks the implementation of the new ERP system that the company hired, grouping all the previous balances as an initial inventory, while the second is the last record delivered by the company. On the other hand, the items used for the analysis are those that are part of the inventory of "basic equipment", because they are one of the items that Agrosavia most requests to carry out the main tasks within the administrative areas, as well as the basic tasks in laboratories. This classification does not take into account the individual demand for each product or those materials that laboratories use in specific projects, so they can be considered as supplies or “spare parts” of the main “production line”, which is the development of research services related with agriculture.

The number of items used was 79, because they were the ones that started 2018 with initial inventory. Each of these item has a specific and unique SKU (Stock Keeping Unit), which acts as an identifier of what and where is the item in the whole inventory system of the company. All the entries, transfers and exits found within each of the storages (main, Cathedral and the eight laboratories) were grouped to review the behavior of each of the items. Subsequently, the exits are taken as the demand for the items and aggregated in a low frequency time series, i.e., from daily to monthly data. In the same way, the number of zeros between periods and the months that are in demand is established, resulting in the following matrix (See Appendix A to see the whole data):

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Dec-17 | Jan-18 | Feb-18 | Mar-18 | Apr-18 | May-18 | Jun-18 | … | Total | Zeros | Months with exits |
| BIP001271 | 0 | 10 | 16 | 13 | 22 | 49 | 28 | … | 686 | 4 | 33 |
| BIP003819 | 0 | 0 | 36 | 0 | 36 | 31 | 3 | … | 1095 | 8 | 29 |
| BIP001998 | 0 | 1 | 8 | 0 | 7 | 7 | 4 | … | 838 | 8 | 29 |
| BIP002015 | 0 | 1 | 1 | 0 | 7 | 16 | 0 | … | 2098 | 8 | 29 |
| BIP002010 | 0 | 0 | 5 | 0 | 8 | 6 | 0 | … | 125 | 11 | 25 |
| BIP002898 | 0 | 0 | 0 | 0 | 0 | 6 | 5 | … | 96 | 15 | 22 |
| BIP001266 | 0 | 0 | 13 | 0 | 3 | 0 | 0 | … | 283 | 16 | 21 |
| BIP007983 | 0 | 0 | 9 | 4 | 7 | 0 | 0 | … | 94876 | 18 | 19 |
| BIP002983 | 0 | 0 | 0 | 0 | 10 | 6 | 0 | … | 304 | 19 | 18 |
| BIP005889 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | … | 99 | 20 | 17 |
| BIP008645 | 0 | 1 | 3 | 0 | 15 | 0 | 0 | … | 4829 | 20 | 17 |
| BIP005467 | 0 | 0 | 100 | 0 | 0 | 0 | 0 | … | 56101 | 20 | 17 |
| BIP008920 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | … | 4875 | 19 | 18 |
| BIP009107 | 0 | 0 | 0 | 0 | 0 | 170 | 50 | … | 1169 | 21 | 16 |
| BIP006806 | 0 | 0 | 0 | 1 | 0 | 6 | 0 | … | 41 | 22 | 15 |
| BIP005887 | 0 | 0 | 2 | 0 | 10 | 52 | 0 | … | 201 | 21 | 16 |
| … | … | … | … | … | … | … | … | … | … | … | … |
| BIP001964 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | … | 29 | 28 | 9 |
| BIP008771 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | … | 25 | 35 | 2 |

Table Monthly demand of Agrosavia’s

## 3.4. Problem Formulation

Due to the lack of application of forecasting methods to the demand of the items belonging to “basic equipment: that Agrosavia has, the first step to develop precise methods is to classify the type of demand that these items have. In this way, the ADI (Average Inter-Demand Interval) and the Coefficient of Variation are calculated, applying the following equations respectively:

(5)

(6)

Where:

* *N*: Number of non-zero demand interval
* *ti:* time period between two consecutive demand periods
* *di:* demand value at time i of the item
* : average demand of the item

It is important to highlight that, to identify the type of demand that the Agrosavia inventory system has, only the average of the periods that have demand is taken into account and not the total periods, unlike the normal variation coefficient. This same situation can be identified in the calculation of the Average Inter-Demand Interval (ADI).

Figure Demand pattern of the items in Agrosavia

Thus, as can be seen in the figure 11, most of the items show a tendency to have intermittent demand (quadrant 4) and lumpy demand (quadrant 1). This is shown, because these items have an ADI higher than 1.32, that is, the average periodicity in which demand occurs is more than 1.32 periods or, in this case, months. This, when contrasting with the models of Syntetos et. al (2005), could justify the application of the method that Syntetos & Boylan propose (2005). Based on the above, Figure 12 shows the number of items that belong to each of the types of demand. It is important to note that, from the beginning, no item of the company has a constant demand, so no evidence of Smooth Demand will be shown.

Figure Number of items by type of demand in Agrosavia

To better illustrate how the demand types of some of the items are observed, Figures 13, 14 and 15 show the erratic, intermittent and lumpy patterns, respectively.

Figure BIP001271 demand pattern (erratic demand)

Figure 13 shows that there is a much more constant periodicity of demand, but the quantities of this are much more irregular, having some high peaks in May 2018, April and August 2019 and October and November 2020, and very small demands. in February and June 2019 and in June, August and September 2020 (these last two months with zero demand)

Figure BIP008013 demand pattern (intermittent demand)

Although the first months of figure 14 do not show any demand, in the subsequent months there is only an irregular peak, in December 2018, and more regular consumption in the following months. However, having a more stable demand in quantities, but less inconsistent in periods of occurrence, the characteristic intermittency of this pattern of demand is observed.

Figure BIP005887 demand pattern (Lumpy demand)

Finally, figure 15 shows two very high peaks of quantities, May 2018 and July 2019, but with quite low consumption in the previous, intermediate and subsequent months. Similarly, the occurrence of these two peaks is almost a year, for which the irregularity in the periods of demand is also evident. Knowing the behavior of the demand of the Agrosavia items, in part 3.5 the application of the chosen models to forecast these slow-moving demands will be expanded, as well as the error measurement mechanisms, so that in chapter 4 it will be possible to discuss about the accuracy of each model and conclude which is most beneficial to the company.

## 3.5 Application of forecasting methods for slow moving items

According to the number of items that were presented to analyze their demand patterns, some of them did not have enough data within the time series, i.e., they had 1, 2 or 3 demand periods, so it was decided not to use them within the application of forecasting methods for slow moving items. Thus, of the 79 initial items, 59 of them were taken to be analyzed by the Croston method (Croston, 1972), the SBA (Syntetos & Boylan, 2005), the Aggregate-Disaggregate Intermittent Demand Approach or ADIDA (Nikolopoulos et al, 2011) and the Multiple Aggregation Prediction Algorithm or MAPA (Kourentzes et al. 2014). Similarly, as seen in Figure 3, Boylan et al. (2008) had proposed the application of the Croston method and the SBA, depending on the type of demand that each of the analyzed items will present. However, in this study all the methods will be taken into account, to analyze which of them performs better in the entire Agrosavia system.

As mentioned, the Croston method was the pioneer in establishing a way to forecast items with irregularities in size and periodicity of their demands. In this way, Croston, based on Exponential Smoothing, decided to separate the components of this method in the demand size and the interdemand interval. Its modifications have focused on the reduction of the bias that this initial method presented, mainly taking two previous periods instead of one (Vinh, 2005), or in the approximation of the calculation of the average demand through the use of the smoothing constants (Syntetos & Boylan 2005 and Shale et al. 2006).

It is important to highlight that, for the application of these methods, the mathematical rigor that must be followed is important. In this way, calculations and predictions were carried out using the R programming language, which is quite accurate for statistical analysis. Additionally, there is a library designed by Kourentzes and Petropoulos (2016) called “*tsintermittent*”, specialized in forecast of intermittent time series, where the functions to be analyzed are already parameterized. In this way, it is important to show how each one of them works.

Using the crost () function of the previously mentioned library, the forecast can be obtained using the Croston method. What this function performs is to take the demand sizes that are different from 0, as well as the periods in which that demand occurs and each one is predicted by means of Exponential Smoothing. However, it differs from this method, as it mixes the two parameters as a whole. To execute it, it is necessary to specify the number of periods to be predicted, so for this study, 5 months will be established. Similarly, it is important to specify the value of the exponential smoothing, which will be 0.15, according to what was recommended in Syntetos & Boylan (2001) and Teunter & Duncan (2009). In addition, other 4 values for alpha will be executed, in order to diversify the analysis and consider more options for the final recommendation.

With the modifications, the process is similar, because it is the same Croston method, but adjusted to each of the concepts that they establish. Thus, to apply the approximation proposed in the SBA model, the type of modification must be specified in the crost () function, in this case 'sba'. The same occurs with the Shale-Boylan-Johnston (2006) correction, 'sbj'.

On the other hand, the library "*tsintermittent*" also includes a useful function to facilitate obtaining ADIDA. As explained in chapter two, this method adds a high-frequency time series into a low-frequency time series, then is forecasted, and this prediction is disaggregated into the original time series. Similarly, to find the "optimal" level of aggregation for the time series, Nikolopoulos et al. (2011) explain that lead time should be used plus a review period. However, as the lead time of the items in this study is not known, there will be a lead time of 1, so the temporary aggregation will be done bimonthly. Nevertheless, to implement the MAPE method, the process is very similar with ADIDA, but offering a much broader aggregation, testing several temporary aggregations simultaneously, resulting in the one that is more optimal for the original series. In this way, the imapa () function can be executed for the two scenarios outlined above: if the periodicity of the series is specified ('minumumAL =' parameter) and the "optimal" aggregation ('maximumAL =' parameter), the function will show the results for the ADIDA method. Otherwise, if no minimums or maximums are revealed, the function will calculate the MAPE method, by means of the actual optimal aggregation that it has found.

Finally, to evaluate the reliability of the forecasts made with each of the methods, 3 types of errors will be calculated: the Mean Error (ME), which will serve to identify the level of bias obtained by the forecast, the Mean Absolute Error (MAE), important to know how much variability the errors present, and the RMSE, which will facilitate the task of comparing the errors of the different models, demonstrating which of them present better precision. It is important to clarify that, due to the amount of zeroes within the time series, the calculation of MAPE will not be necessary, because being a proportion, it would give an undefined value (Montgomery et al. 2016)

Summarizing, the whole method applied to the data will be designated as it is seen in the Figure 16:

Classification of item’s demand patterns

Application of slow-moving demand forecasting methods

Measurement of the accuracy of the forecasting methods

Selection of the best forecasting method for each classification

Figure Scheme for evaluating the forecasting process of Agrosavia’s items

# CHAPTER 4 RESULTS AND DISCUSSION

First, it is important to highlight that the results obtained will be grouped according to the forecasting methods used and the 4 types of items with intermittent demand that have been described above and that comply with the parameters established by Syntetos & Boylan (2005). In this way, to facilitate the understanding of the outcome, an item will be taken from each of the types of intermittent demand identified and the analysis will be carried out, so that it will be generalized for the other items found in that classification. However, the application of forecasting methods and the measurement of errors was applied to all items provided by the evaluated company.

## 4.1 Croston’s and Exponential Smoothing Methods

For the development of these methods, five experiments were carried out evaluating different alphas per item, so that a much more diverse comparison could be made, not only evaluating the type of method, but also the consequences of changing the smoothing constant. In this way, the following values were chosen:

* α = 0.15, as it was recommended by Syntetos and Boylan (2001)
* α = 0.1, in order to review if values inferior as 0.15 would perform better, as Syntetos & Boylan (2001) said.
* α = 0.5, as a maximum value, to see how high values affect the methods and,
* α = “optimal”, which is calculated automatically by the function. This one can vary according to every method.

### 4.1.1 Erratic Items

Since, during the classification of items, none were found that had the type "smooth", the first classification of items to analyze is "erratic". For this situation, item BIP001271 was taken, which presents a relatively constant demand, except for 3 periods, but given the application of the ADI and the Coefficient of Variation, its demand is erratic. This can be demonstrated in Figure 17.

Figure Item BIP001271 demand behavior

One of the main characteristics that this type of item presents is that they usually have an irregular demand size (since it does not have a constant or moderately similar demand), but its occurrence is much more constant. When applying the Croston and Exponential Smoothing methods, it is important to note that the forecast obtained is an average of the demand that will occur in the following periods, so it will be a constant (Croston, 1972; Vinh, 2005; Syntetos & Boylan, 2005; Shale et al., 2006; Teunter et al, 2011). Thus, Table 2 shows the smoothed results of each period using α = 0.1, as well as the forecast of the average demand per period (the complete table can be found in the annex)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | BIP001271 | SES\_bip1271 | cros\_smooth | SBA\_smooth | SBJ\_smooth |
| Dec-17 | 0 | 0 |  |  |  |
| Jan-18 | 10 | 0 |  |  |  |
| Feb-18 | 16 | 1 | 16.7873012 | 17.18181751 | 17.18182087 |
| Mar-18 | 13 | 2.5 | 16.75918329 | 17.11103829 | 17.10953779 |
| Apr-18 | 22 | 3.55 | 16.615703 | 16.92931923 | 16.92657029 |
| May-18 | 49 | 5.395 | 16.83475497 | 17.09086039 | 17.08586789 |
| Jun-18 | 28 | 9.7555 | 18.22586507 | 18.36493469 | 18.35458103 |
| Jul-18 | 15 | 11.57995 | 18.67401845 | 18.74252032 | 18.72926282 |
| Aug-18 | 7 | 11.921955 | 18.4959172 | 18.52474145 | 18.51021313 |
| Sep-18 | 13 | 11.4297595 | 17.9083697 | 17.91783246 | 17.90310529 |
| Oct-18 | 16 | 11.58678355 | 17.64461134 | 17.61863688 | 17.6028628 |
| Nov-18 | 29 | 12.0281052 | 17.55194887 | 17.48236342 | 17.46510578 |
| … | … | … | … | … | … |
| Oct-20 | 41 | 12.71635803 | 16.40118523 | 15.7257789 | 15.68779738 |
| Nov-20 | 60 | 15.54472223 | 15.75842652 | 15.0811569 | 15.04364941 |
| Fore. 1 |  | 19.99025 | 19.30151 | 18.43823 | 18.39108 |
| Fore. 2 |  | 19.99025 | 19.30151 | 18.43823 | 18.39108 |
| Fore. 3 |  | 19.99025 | 19.30151 | 18.43823 | 18.39108 |
| Fore. 4 |  | 19.99025 | 19.30151 | 18.43823 | 18.39108 |
| Fore. 5 |  | 19.99025 | 19.30151 | 18.43823 | 18.39108 |

Table Smoothing and forecast of item BIP001271

Figure Croston's and Expo. Smooth. forecasts of item BIP001271 with α = 0.1

Similarly, Figure 18 shows the graphic behavior of this procedure. In it, it can be seen that the Exponential Smoothing takes the previous values to perform its procedure (this will be seen in every experiment using these methods). However, due to the nature of the Croston method, in which it is only updated when there is demand, and given that all the items used within this study have initial demand of 0, the first value is not taken into account due to the uncertainty that exists related to the previous period in which there was demand other than zero (Petropoulos et al., 2016).

However, detailing the figure, it is possible to affirm that, although the Exponential Smoothing takes the demand values from the beginning, it tends to take time to smooth the data, so that its bias may be more accentuated compared to the Croston methods. Similarly, it seems that the latter show a fairly similar behavior, in such a way that it seems that there are imperceptible changes in their results. Given this situation, it is necessary to review the errors generated during the forecast, which appear in Table 3.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | SES | Croston | SBA | SBJ |
| ME | 5.55 | 1.48 | 1.81 | 1.84 |
| MAE | 13.81 | 12.06 | 11.97 | 11.96 |
| RMSE | 17.46 | 15.58 | 15.57 | 15.58 |
| Error size | 3.65 | 3.52 | 3.61 | 3.61 |
| RMSE variation | | | | |
| SES |  | 12.05% | 12.13% | 12.12% |
| Croston | -10.76% |  | 0.06% | 0.06% |
| SBA | -10.81% | -0.06% |  | -0.01% |
| SBJ | -10.81% | -0.06% | 0.01% |  |
| MAE variation | | | | |
| SES |  | 14.54% | 15.41% | 15.47% |
| Croston | -12.70% |  | 0.76% | 0.81% |
| SBA | -13.35% | -0.75% |  | 0.05% |
| SBJ | -13.40% | -0.80% | -0.05% |  |

Table Errors of item BIP001271 forecast with α = 0.1

As can be seen, there is a marked difference in the calculated MEs. This shows that there is a significant amount of bias in the forecast generated by Exponential Smoothing, which argues for the lack of precision that this method has, compared to Croston's. This is also demonstrated when calculating the MAE, with an increase of around 15%, and, ultimately, the RMSE, which is greater than the others around 12%. However, it is interesting to see that the Croston method has lower bias than the SBA and SBJ methods, although its RMSE and MAE present very close values, since their differences do not even reach 1%. In this way, it is possible to affirm that by having such small differences within their MAE and RMSE, the three Croston methods present a similar precision by having an alpha of 0.1, but they are much more precise than the Exponential Smoothing.

Figure Croston's and Expo. Smooth. forecasts of item BIP001271 with α = 0.15

On the other hand, by increasing the alpha to 0.15, it can be seen in Figure 19 that, apparently, the Exponential Smoothing presents an improvement, by reducing its difference with the data presented. In the same way, the results of the Croston method can be better distinguished by an increase in the smoothing of the forecast, so increasing the alpha a little negatively affects the forecast accuracy of this method, at least for items with erratic demand. In the same way, when analyzing the errors of this iteration, Table 4 reiterates the improvement of the Exponential Smoothing, but it is still less effective, since its MAE is still greater than the Croston method by a little more than 11% (about 3 % improvement versus alpha = 0.1), and about 13% with the SBA and SBJ methods. Likewise, the former method presents a reduction of the RMSE, about 8% in relation to the other methods, but it is still high enough to demonstrate its ineffectiveness. On the other hand, although the Croston-type methods perform better, it is evident that the original Croston loses some efficiency against the other two (almost 2% for the MAE and although less than 1% for the RMSE). This justifies what was said by Syntetos and Boylan (2005) and by Teunter and Duncan (2009), related to the best performance of this method when the alpha used is a maximum of 0.15.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | SES | Croston | SBA | SBJ |
| ME | 4.08 | 1.47 | 2.14 | 2.16 |
| MAE | 13.66 | 12.26 | 12.05 | 12.04 |
| RMSE | 17.11 | 15.80 | 15.78 | 15.77 |
| Error size | 3.45 | 3.54 | 3.73 | 3.74 |
| RMSE variation | | | | |
| SES |  | 8.28% | 8.42% | 8.45% |
| Croston | -7.65% |  | 0.13% | 0.16% |
| SBA | -7.77% | -0.13% |  | 0.03% |
| SBJ | -7.79% | -0.16% | -0.03% |  |
| MAE variation | | | | |
| SES |  | 11.42% | 13.37% | 13.46% |
| Croston | -10.25% |  | 1.75% | 1.83% |
| SBA | -11.79% | -1.72% |  | 0.08% |
| SBJ | -11.86% | -1.80% | -0.08% |  |

Table Errors of item BIP001271 forecast with α = 0.15

In reference to alphas greater than 0.15, it can be seen in Figure 20 that both the Exponential Smoothing and the Croston method tend to converge, except at times when demand is equal to 0 (periods of August and September 2020), due to the nature of the method. On the other hand, all the methods tend to resemble the behavior of the original data, although it is perceived that an increase in alpha reduces the precision of all the methods, especially the SBJ.

Figure Croston's and Expo. Smooth. forecasts of item BIP001271 with α = 0.5

When reviewing the errors, the inaccuracy of the methods is evident as the alpha increases. Table 5 shows a small improvement in the forecast errors offered by the Exponential Smoothing, with variations that do not even reach 1% compared to the Croston and SBA methods. However, it is surprising that the SBJ method is the least accurate, when talking about the RMSE, since it has an increase of approximately 1.5% to almost 3%, compared to the other methods. It is interesting to review that the SES presents a better behavior of the RMS, but its MAE is not the best, as it increases in comparison with the Croston methods, while the SBJ method presents a totally opposite behavior. Thus, when the difference between the RMSE and the MAE is made, there is a much wider variation in the latter method than in the SES, even this method is the one that offers a more attenuated variation of errors, indicating that, although its mean errors are still usually high, the individual ones tend to vary to a lesser extent. However, when comparing the 4 methods, Croston is the one that shows the most favorable performance, having the lowest RMSE, as well as individual errors with lower variance.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | SES | Croston | SBA | SBJ |
| ME | 2.29 | 1.06 | 5.15 | 6.53 |
| MAE | 13.49 | 12.91 | 12.23 | 12.25 |
| RMSE | 16.98 | 16.78 | 16.92 | 17.26 |
| Error size | 3.49 | 3.87 | 4.69 | 5.01 |
| RMSE variation | | | | |
| SES |  | 1.19% | 0.34% | -1.63% |
| Croston | -1.18% |  | -0.84% | -2.79% |
| SBA | -0.34% | 0.85% |  | -1.96% |
| SBJ | 1.66% | 2.87% | 2.00% |  |
| MAE variation | | | | |
| SES |  | 4.47% | 10.29% | 10.12% |
| Croston | -4.28% |  | 5.57% | 5.41% |
| SBA | -9.33% | -5.28% |  | -0.16% |
| SBJ | -9.19% | -5.13% | 0.16% |  |

Table Errors of item BIP001271 forecast with α = 0.5

Finally, the last iteration performed with the optimums suggested by the function, show smoother results than in the iterations using large alphas (0.5), since, as will be seen in Table 7, it uses alphas smaller than 0.1 for the case of Croston's methods. However, for the case of Exponential Smoothing, the alpha used is approximately 0.32, which indicates that this model generates more optimal forecasts using larger alphas, contrary to the former methods, as shown in Figure 21.

Figure Croston's and Exp. Smooth. forecasts of item BIP001271 with α = “opt.”

Based on this and on Table 6, it can be seen that, in short, the alphas calculated for Croston's methods, which are less than 0.1, show a considerable improvement at higher alphas, unlike Exponential Smoothing, which responds better to increases in this parameter. Similarly, when comparing the "optimal" results, it can be seen that there is an improvement in the accuracy of the SES, specifically the RMSE, which indicates that, as the alpha increases, for this method, its effectiveness increases, since when comparing the results of the 4 tables, it starts with a maximum variation of about 12% with alpha = 0.1, goes through a variation of about 7% with optimal alpha of 0.32 and ends with almost 1% using an alpha of 0.5. However, although the MAE and RMSE in their last iterations seem to increase and decrease respectively, the size of SES errors offers a slight improvement in the case of optimal alpha, which suggests that individually its errors behave lighter, hence its prediction is more accurate. However, when generalizing the results of the four methods, the Exponential Smoothing, either with small or larger values of alpha, generates the least accurate results when forecasting erratic demand.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | SES | Croston | SBA | SBJ |
|  | 0.3248 | 0.0975 | 0.0832 | 0.0914 |
| ME | 2.68 | 2.45 | 2.49 | 2.40 |
| MAE | 13.58 | 12.24 | 12.31 | 12.14 |
| RMSE | 16.88 | 15.79 | 15.83 | 15.73 |
| Error size | 3.30 | 3.55 | 3.52 | 3.59 |
| RMSE variation | | | | |
| SES (0.1) | -3.31% | 8.34% | 8.41% | 8.40% |
| SES (0.15) | -1.30% | 6.87% | 7.01% | 7.04% |
| Croston (0.1) | -9.58% | 1.31% | 1.38% | 1.37% |
| Croston (0.15) | -7.71% | -0.07% | 0.07% | 0.09% |
| SBA (0.1) | -9.33% | 1.60% | 1.67% | 1.66% |
| SBA (0.15) | -7.45% | 0.22% | 0.35% | 0.38% |
| SBJ (0.1) | -9.91% | 0.95% | 1.02% | 1.01% |
| SBJ (0.15) | -8.04% | -0.42% | -0.29% | -0.26% |
| MAE variation | | | | |
| SES (0.1) | -1.68% | 12.61% | 13.47% | 13.53% |
| SES (0.15) | -0.56% | 10.80% | 12.73% | 12.83% |
| Croston (0.1) | -11.41% | 1.48% | 2.25% | 2.30% |
| Croston (0.15) | -10.39% | -0.16% | 1.59% | 1.67% |
| SBA (0.1) | -10.89% | 2.07% | 2.85% | 2.90% |
| SBA (0.15) | -9.87% | 0.43% | 2.18% | 2.26% |
| SBJ (0.1) | -12.09% | 0.70% | 1.46% | 1.51% |
| SBJ (0.15) | -11.08% | -0.93% | 0.80% | 0.89% |

Table Errors of item BIP001271 forecast with α = “optimal”

On the other hand, it is surprising to note that what Syntetos & Boylan (2001) explained about the effectiveness of the Croston method with alphas lower than 0.15 is partially true. In other words, it is fulfilled, but not when its alpha decreases less than 0.1. As shown in Tables 3, 4 and 6, the best performance of this method is when using alpha = 0.1, since its MAE and RMSE is the lowest, as well as the variability of its individual errors. The same happens with the SBJ method, although it presents a tendency to increase its errors, hence to have a less accurate prediction, when alpha increases, although its behavior improves at optimal alphas. Finally, the method that presents the most favorable stability during all iterations is the SBA. It may be that during the optimal alphas it does not present the best behavior, since its errors are the highest with reference to the Croston methods, besides that all the methods present differences in the number assigned to alpha, but a remarkable improvement can be observed when comparing the results of its other iterations, evidenced in tables 3, 4 and 5. Thus, it is possible to admit that the method that works best with the erratic demand items, when comparing the alphas, is the SBA, although it is advisable to use an alpha equal to or higher than 0.1.

Similarly, during the analysis, the smoothing of the inter-demand intervals was taken into account, since this could predict when the next demand would occur or how many periods later it might occur. However, it is important to clarify that, although Croston type methods do not take this value individually to calculate their forecast, it could be taken into account in this analysis to identify how accurate the reading of this parameter could be. Thus, Table 7 shows the results of the smoothing of this parameter in all Croston methods, since the Exponential Smoothing does not make use of it for its execution. (For complete data, please see the annex)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| α | 0.1 | | | 0.15 | | | 0.5 | | | opt | | |
|  | **Cro.** | **SBA** | **SBJ** | **Cro.** | **SBA** | **SBJ** | **Cro.** | **SBA** | **SBJ** | **Cro.** | **SBA** | **SBJ** |
| 17-Dec |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-Jan |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-Feb | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 2.62 | 2.33 | 1.00 | 1.00 | 1.00 |
| 18-Mar | 2.80 | 2.80 | 2.80 | 2.70 | 2.70 | 2.70 | 2.00 | 1.81 | 1.66 | 1.00 | 1.00 | 1.00 |
| 18-Apr | 2.62 | 2.62 | 2.62 | 2.45 | 2.45 | 2.45 | 1.50 | 1.40 | 1.33 | 1.00 | 1.00 | 1.00 |
| 18-May | 2.46 | 2.46 | 2.46 | 2.23 | 2.23 | 2.23 | 1.25 | 1.20 | 1.17 | 1.00 | 1.00 | 1.00 |
| 18-Jun | 2.31 | 2.31 | 2.31 | 2.04 | 2.04 | 2.04 | 1.13 | 1.10 | 1.08 | 1.00 | 1.00 | 1.00 |
| 18-Jul | 2.18 | 2.18 | 2.18 | 1.89 | 1.89 | 1.89 | 1.06 | 1.05 | 1.04 | 1.00 | 1.00 | 1.00 |
| 18-Aug | 2.06 | 2.06 | 2.06 | 1.75 | 1.75 | 1.75 | 1.03 | 1.03 | 1.02 | 1.00 | 1.00 | 1.00 |
| 18-Sep | 1.96 | 1.96 | 1.96 | 1.64 | 1.64 | 1.64 | 1.02 | 1.01 | 1.01 | 1.00 | 1.00 | 1.00 |
| 18-Oct | 1.86 | 1.86 | 1.86 | 1.54 | 1.54 | 1.54 | 1.01 | 1.01 | 1.01 | 1.00 | 1.00 | 1.00 |
| 18-Nov | 1.77 | 1.77 | 1.77 | 1.46 | 1.46 | 1.46 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 18-Dec | 1.70 | 1.70 | 1.70 | 1.39 | 1.39 | 1.39 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 19-Jan | 1.63 | 1.63 | 1.63 | 1.33 | 1.33 | 1.33 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| … | … | … | … | … | … | … | … | … | … | … | … | … |
| 19-Mar | 1.51 | 1.51 | 1.51 | 1.24 | 1.24 | 1.24 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 20-Sep | 1.08 | 1.08 | 1.08 | 1.02 | 1.02 | 1.02 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 20-Oct | 1.08 | 1.08 | 1.08 | 1.02 | 1.02 | 1.02 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 20-Nov | 1.28 | 1.28 | 1.28 | 1.31 | 1.31 | 1.31 | 2.00 | 2.00 | 2.00 | 1.00 | 1.00 | 1.00 |

Table Smoothed inter-demand interval of item BIP001271 (erratic demand)

As can be seen, for the cases in which the alpha is not optimal, the inter-demand interval forecast shows high values, from 3 to 2 approximately, in reference to the fact that it takes the values prior to the first demand and counts them as periods in which there is no demand, as indicated by (Petropoulos et al., 2016). However, when taking this parameter as a possible indicator to forecast the next period in which a demand will occur, given that the erratic type items are constant in their occurrence, but variable in the size of the demand, the period between demands is mostly at 1 (every period demand will occur). However, the periods in which there is no demand (August 2020 and September 2020), although it tends to reduce its forecast, still tends to 1, so it is not very accurate in forecasting periods in which demand is zero. Therefore, for erratic items, forecasting the next period in which demand occurs, by revising the inter-demand interval, may lead it to show that all future periods will have demand.

### 4.1.2. Intermittent items

The next classification to be analyzed is that of items with intermittent demand. In this case, item BIP008013 was taken, which presents the necessary characteristics to be classified within this group, i.e., its occurrence is much rarer (so the number of zeros within the time series will be much higher than in the erratic items), but the size of its demand varies with less intensity. Practically, they are the antithesis of erratic demand items. This can be seen in Figure 22.

Figure Item BIP001271 demand behavior

On the other hand, the forecasting and smoothing of the data will be presented in the annexes. Thus, starting the first iteration with alpha = 0.1, the behavior of its application can be observed in Figure 23. It shows the same phenomenon as in the erratic items, where the Exponential Smoothing takes all the initial data and the Croston methods start when there is demand. Similarly, it can be observed that the most disparate smoothing is that of Exponential Smoothing, while the Croston methods tend to generate a much more even smoothing, it is even difficult to notice any difference between them, so it is necessary to go into detail and analyze their errors.

Figure Croston's and Exp. Smooth. forecasts of item BIP008013, alpha = 0.1

Thus, Table 8 shows the errors of this iteration, indicating that, as observed in the erratic items, the Exponential Smoothing shows the highest errors, mainly in the RMSE, which usually penalizes the errors when they are higher. This is logical in the sense that, first, the demand size of this item is much higher and, second, the variability of the demand changes much more than with the first item (given that there are more zeros during the time series). Thus, when evaluating this type of item with this method, the result tends to be much broader. However, it still indicates that, compared to Croston's methods, Exponential Smoothing is less accurate, with a difference of about 3%. This is also confirmed by checking the MAE, which, although has smaller variations, is still higher (between 0.29% and 0.39%). Similarly, individual errors tend to be higher than with the other methods. On the other hand, when comparing between Croston methods, they have very slight differences, although the original method benefits a little less from this alpha by having differences of 0.04% in its MAE and approximately 0.10% in its RMSE. Finally, between the two modifications (SBA and SBJ) there are quite similar results, but they are still the most accurate in their forecasting, since even the size of their mean errors is the lowest, as well as their MAE and RMSE. It seems to indicate that the corrections applied by Syntetos & Boylan (2005) and Shale, Boylan and Johnston (2005), are much more effective in this type of items and that, when used with low alphas (or at least 0.1), they present better results.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | SES | Croston | SBA | SBJ |
| ME | 223.46 | 301.13 | 303.39 | 303.51 |
| MAE | 759.26 | 757.05 | 756.36 | 756.32 |
| RMSE | 1312.32 | 1272.41 | 1271.94 | 1271.91 |
| Error size | 553.06 | 515.36 | 515.58 | 515.59 |
| RMSE variation | | | | |
| SES |  | 3.14% | 3.17% | 3.18% |
| Croston | -3.04% |  | 0.04% | 0.04% |
| SBA | -3.08% | -0.04% |  | 0.00% |
| SBJ | -3.08% | -0.04% | 0.00% |  |
| MAE variation | | | | |
| SES |  | 0.29% | 0.38% | 0.39% |
| Croston | -0.29% |  | 0.09% | 0.10% |
| SBA | -0.38% | -0.09% |  | 0.01% |
| SBJ | -0.39% | -0.10% | -0.01% |  |

Table Errors of item BIP008013 forecast with α = 0.1

When reviewing the iteration using alpha = 0.15, it can be observed the similarity of using low alphas when applying these forecasting methods, as shown in Figure 24. That is, the Exponential Smoothing values are going to be taken from the beginning, while the Croston values start from November 2018, beginning to forecast from the following period. Similarly, the latter methods continue to show a behavior similar to that of the use of alpha = 0.1, while in Exponential Smoothing an accentuation is observed, i.e., it tries to match, little by little, the time series, although a sufficiently defined smoothing is still observed.

Figure Croston's and Exponential Smoothing forecasts of item BIP008013, alpha = 0.15

Thus, when analyzing the errors provided by Table 9, it can be observed that Exponential Smoothing, within its MAE, presents a better forecast than the other methods, while its RMSE is higher. This is surprising, because if the size of individual errors among all the methods is reviewed, this method still shows the highest. However, it is remarkable to observe that, by increasing the alpha by 0.05, the performance of Exponential Smoothing improves when applied to items of an intermittent nature. On the other hand, when evaluating Croston's methods, a behavior similar to alpha = 0.1 can be observed, since again Croston shows slightly higher errors than SBA and SBJ (0.06% and 0.07% respectively) in its RMSE and around 0.20% in its MAE, although the difference between their individual error sizes is the highest among all the methods. Finally, when reviewing the SBA and SBJ methods, it can be seen that again they do not change much with each other, which continues to justify that, as seen with alpha = 0.1, these two methods work similarly when they have low alphas, in this case less than or equal to 0.15.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | SES | Croston | SBA | SBJ |
| ME | 145.05 | 313.35 | 317.82 | 318.20 |
| MAE | 754.68 | 760.09 | 758.64 | 758.52 |
| RMSE | 1301.66 | 1278.07 | 1277.25 | 1277.19 |
| Error size | 546.98 | 517.98 | 518.61 | 518.67 |
| RMSE variation | | | | |
| SES |  | 1.85% | 1.91% | 1.92% |
| Croston | -1.81% |  | 0.06% | 0.07% |
| SBA | -1.87% | -0.06% |  | 0.01% |
| SBJ | -1.88% | -0.07% | -0.01% |  |
| MAE variation | | | | |
| SES |  | -0.71% | -0.52% | -0.51% |
| Croston | 0.72% |  | 0.19% | 0.21% |
| SBA | 0.52% | -0.19% |  | 0.02% |
| SBJ | 0.51% | -0.21% | -0.02% |  |

Table Errors of item BIP008013 forecast with α = 0.15

Now, by increasing the alpha to a much larger value, i.e. 0.5, a much higher approximation to the original data can be evidenced, mainly from the Exponential Smoothing, so that its mean errors will be much lower than in the previous iterations. On the other hand, although the other methods still present the same characteristics that have represented them, the smoothing that they show is no longer so evident and they tend to resemble the behavior of the Exponential Smoothing. This is evidenced in Figure 25.

Figure Croston's and Exp. Smooth. forecasts of item BIP008013, alpha = 0.5

Reviewing the errors of this iteration in Table 10, we observe a significant decrease in the mean errors of the Exponential Smoothing, as mentioned above, but both its RMSE and MAE are much higher when compared to the other methods (except for the RMSE of the Croston method, against which it shows a slight improvement, although it is still higher). Thus, Exponential Smoothing, by raising alpha to a much higher value, generates a much more inadequate forecast than the methods developed to forecast slow-moving demand. On the other hand, the Croston method does not show a more accurate option than its modifications, since its RMSE and MAE are still higher than these (about 5% and 8% respectively), and it also presents the largest size of individual errors. On the other hand, the two modifications show a slightly more evident difference to the previous iterations, giving the SBJ method as somewhat more accurate than the SBA, since its RMSE is lower by 0.45% and its MAE by 0.70%, without detracting from the fact that the SBA also shows a much better performance. However, it is important to emphasize that using this high alpha does not show more accurate results than if it were lower than 0.15. On the contrary, it is detrimental to the forecasting, both for the Exponential Smoothing and the other methods, its application.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | SES | Croston | SBA | SBJ |
| ME | 45.35 | 138.56 | 283.01 | 329.14 |
| MAE | 815.04 | 785.99 | 759.53 | 754.19 |
| RMSE | 1378.39 | 1357.48 | 1319.56 | 1313.67 |
| Error size | 563.35 | 571.49 | 560.03 | 559.48 |
| RMSE variation | | | | |
| SES |  | 1.54% | 4.46% | 4.93% |
| Croston | -1.52% |  | 2.87% | 3.34% |
| SBA | -4.27% | -2.79% |  | 0.45% |
| SBJ | -4.70% | -3.23% | -0.45% |  |
| MAE variation | | | | |
| SES |  | 3.70% | 7.31% | 8.07% |
| Croston | -3.56% |  | 3.48% | 4.22% |
| SBA | -6.81% | -3.37% |  | 0.71% |
| SBJ | -7.47% | -4.05% | -0.70% |  |

Table Errors of item BIP008013 forecast with α = 0.5

Finishing with the alpha analysis, using the "optimum" for each case, Figure 26 shows again a smoothing similar to that found in the alphas less than or equal to 0.15, for all cases, although a little more accentuated in the case of Exponential Smoothing. It is important to emphasize that, as in the case of the erratic items, the alphas obtained for each case are different, so that the effectiveness of their predictions is not ideal. However, Table 11 shows the behavior of each of them.

Figure Croston's and Exp. Smooth. forecasts of item BIP008013, α = “opt.”

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | SES | Croston | SBA | SBJ |
| α | **0.1827** | **0.125202** | **0.1266** | **0.16237** |
| ME | 117.18 | 326.70 | 326.36 | 333.10 |
| MAE | 752.03 | 754.60 | 754.43 | 756.79 |
| RMSE | 1300.08 | 1319.38 | 1319.63 | 1322.54 |
| Error size | 548.05 | 564.78 | 565.20 | 565.75 |
| RMSE variation | | | | |
| SES (0.1) | -0.93% | 2.17% | 2.21% | 2.21% |
| SES (0.15) | -0.12% | 1.72% | 1.79% | 1.79% |
| Croston (0.1) | 0.54% | 3.69% | 3.73% | 3.73% |
| Croston (0.15) | 1.36% | 3.23% | 3.30% | 3.30% |
| SBA (0.1) | 0.56% | 3.71% | 3.75% | 3.75% |
| SBA (0.15) | 1.38% | 3.25% | 3.32% | 3.32% |
| SBJ (0.1) | 0.78% | 3.94% | 3.98% | 3.98% |
| SBJ (0.15) | 1.60% | 3.48% | 3.55% | 3.55% |
| MAE variation | | | | |
| SES (0.1) | -0.95% | -0.66% | -0.57% | -0.57% |
| SES (0.15) | -0.35% | -1.06% | -0.87% | -0.86% |
| Croston (0.1) | -0.61% | -0.32% | -0.23% | -0.23% |
| Croston (0.15) | -0.01% | -0.72% | -0.53% | -0.52% |
| SBA (0.1) | -0.64% | -0.35% | -0.25% | -0.25% |
| SBA (0.15) | -0.03% | -0.74% | -0.55% | -0.54% |
| SBJ (0.1) | -0.33% | -0.04% | 0.06% | 0.06% |
| SBJ (0.15) | 0.28% | -0.43% | -0.24% | -0.23% |

Table Errors of item BIP008013 forecast with α = “optimal”

As can be seen, most of the methods show some discrepancy when comparing the optimal forecasts with those of the previous iterations (with the exception of the alpha equal to 0.5, due to its inaccuracy), because while its RMSE indicates a detrimental accuracy for the optimized results, the MAE shows that the forecast with these alphas has some improvement with respect to the previous iterations. This shows that, mainly, the optimization performed by the function is aimed at reducing the amount of errors calculated by the MAE. Similarly, as mentioned above, since the RMSE gives more weight to high errors and since the difference between the forecast and the actual data is higher due to the number of periods without demand, the result may be slightly out of phase. However, it can be shown that, by iterating with this optimization, both Croston's method and the SBA had improvement in their MAE, i.e., they are more accurate with alphas between 0.1 and 0.15. On the other hand, although its alpha exceeded the 0.15 limit, the SBJ method showed similar accuracy to that of the SBA, but what has been more remarkable is that the Exponential Smoothing, having the highest alpha within this iteration, presents the best MAE and the best RMSE. Even comparing it with the previous iterations, its result is the most optimal, when evaluating this aspect, so that, for intermittent items, given these results, Exponential Smoothing presents a higher accuracy, contrary to what many authors have mentioned.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0.1 | | | 0.15 | | | 0.5 | | | opt | | |
|  | Cro. | SBA | SBJ | Cro. | SBA | SBJ | Cro. | SBA | SBJ | Cro. | SBA | SBJ |
| 17-Dec |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-Jan |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-Feb |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-Mar |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-Apr |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-May |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-Jun |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-Jul |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-Aug |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-Sep |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-Oct |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-Nov |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-Dec | 4.04 | 3.68 | 3.66 | 8.08 | 7.00 | 6.91 | 12.00 | 12.00 | 12.00 | 1.78 | 1.78 | 1.74 |
| 19-Jan | 3.74 | 3.41 | 3.39 | 7.02 | 6.10 | 6.03 | 6.50 | 6.50 | 6.50 | 1.78 | 1.78 | 1.74 |
| 19-Feb | 3.74 | 3.41 | 3.39 | 7.02 | 6.10 | 6.03 | 6.50 | 6.50 | 6.50 | 1.78 | 1.78 | 1.74 |
| 19-Mar | 3.74 | 3.41 | 3.39 | 7.02 | 6.10 | 6.03 | 6.50 | 6.50 | 6.50 | 1.78 | 1.78 | 1.74 |
| 19-Apr | 3.74 | 3.41 | 3.39 | 7.02 | 6.10 | 6.03 | 6.50 | 6.50 | 6.50 | 1.78 | 1.78 | 1.74 |
| … | … | … | … | … | … | … | … | … | … | … | … | … |
| 19-Jun | 3.49 | 3.22 | 3.21 | 5.73 | 5.07 | 5.01 | 3.13 | 3.13 | 3.13 | 1.78 | 1.78 | 1.74 |
| 19-Jul | 3.24 | 3.00 | 2.99 | 5.02 | 4.46 | 4.41 | 2.06 | 2.06 | 2.06 | 1.78 | 1.78 | 1.74 |
| 20-Oct | 2.27 | 2.18 | 2.17 | 2.47 | 2.34 | 2.33 | 1.81 | 1.81 | 1.81 | 1.78 | 1.78 | 1.74 |
| 20-Nov | 2.24 | 2.16 | 2.15 | 2.40 | 2.29 | 2.28 | 1.91 | 1.91 | 1.91 | 1.78 | 1.78 | 1.74 |

Table Smoothed inter-demand interval of item BIP008013 (intermittent demand)

Finally, when reviewing the smoothing of the inter-demand intervals in Table 12, it can be seen that the use of these results as an indicator to predict the next period where demand will occur is somewhat incongruent, in the sense that, although in most cases its initial value is decreasing, indicating that there is demand in those periods, at the times when this indicator increases, the accumulation of periods in which there was no demand would be evident. However, this would not function as an indicator to check when there might be demand. Possibly, it can be used to predict when there is no demand, only if the smoothed value maintains a certain constancy which, as is characteristic of Croston's method, indicates that there is no demand in those periods. In any case, it is not intuitive to use this parameter to check, in the future, how to predict demand or lack of it, and it would have to be evaluated period by period for this information to be useful and that would entail a higher cost.

### 4.1.3 Lumpy demand items

Finally, the last classification of items to be evaluated is that of lumpy items, which is characterized by having the shortcomings of the two types of demands analyzed above, i.e., they have a low occurrence, and the size of their demand varies greatly. Practically, it is the antithesis of the items with smooth demand, which were not treated in this analysis, because none of the supplies taken for this study had this behavior. Thus, Figure 27 shows the behavior of item BIP005887, where it can be visibly appreciated that the periodicity with which its demand occurs is quite ephemeral, since it has 20 months in which it is equal to zero. Additionally, it can be observed that it has quite high peaks, which shows that the size of the demand for this item is significantly higher than in other periods.

Figure Item BIP005887 demand behavior

Regarding the analysis of the forecasts, starting with the use of alpha equal to 0.1, Figure 28 shows an apparent similarity between Croston's methods, as evidenced in the previous applications. Similarly, it is observed that Exponential Smoothing tries to follow the same trend of the above-mentioned methods but suffers more variations due to the peaks and valleys given by the variation of the demand and the periods with demand equal to 0.

Figure Croston's and Exp. Smooth. forecasts of item BIP005887, α = 0.1

Moving on to the analysis of the errors of this forecast in Table 13, it can be observed that the Exponential Smoothing generates the least accurate results, since both its RMSE and MAE have a significant increase compared to the other methods (around 5% and 2%, respectively). However, when comparing the differences between Croston's methods, it is observed that the original is the least accurate, by a slight difference (0.24% for the RMSE and 0.85% for the MAE), while its modifications do not present an apparent difference. Practically, the behavior of these two methods generates the same results, except for some variations in a few additional decimal places, indicating a fairly high accuracy. Similarly, when reviewing the size of the individual errors although the Exponential Smoothing keeps the highest ones, all the methods tend to have the same indicator, which could mean that the variation of their errors is not so high, except, evidently, when looking at the periods with the high peaks.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | SES | Croston | SBA | SBJ |
| ME | 1.11 | 0.07 | 0.15 | 0.15 |
| MAE | 7.13 | 7.06 | 7.00 | 7.00 |
| RMSE | 12.76 | 12.16 | 12.13 | 12.13 |
| Error size | 5.63 | 5.10 | 5.13 | 5.13 |
| RMSE variation | | | | |
| SES |  | 4.95% | 5.20% | 5.20% |
| Croston | -4.72% |  | 0.24% | 0.24% |
| SBA | -4.94% | -0.24% |  | 0.00% |
| SBJ | -4.95% | -0.24% | 0.00% |  |
| MAE variation | | | | |
| SES |  | 1.03% | 1.90% | 1.91% |
| Croston | -1.02% |  | 0.85% | 0.86% |
| SBA | -1.86% | -0.85% |  | 0.01% |
| SBJ | -1.87% | -0.86% | -0.01% |  |

Table Errors of item BIP005887 forecast with α = 0.1

By increasing the alpha to 0.15, as shown in Figure 29, a similar behavior to the previous case can be observed, where the Croston methods show similar forecasts and do not change much, contrary to what happens with the Exponential Smoothing, which presents peaks that are a little more visible and attenuated than with the previous iteration. This might indicate that its forecast, again, would be more inaccurate than that of the previous methods, but that will be seen in Table 14.

Figure Croston's and Exp. Smooth. forecasts of item BIP005887, α = 0.15

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | SES | Croston | SBA | SBJ |
| ME | 0.65 | 0.15 | 0.17 | 0.18 |
| MAE | 7.44 | 7.07 | 7.03 | 7.03 |
| RMSE | 12.87 | 12.23 | 12.19 | 12.19 |
| Error size | 5.43 | 5.16 | 5.16 | 5.17 |
| RMSE variation | | | | |
| SES |  | 5.21% | 5.54% | 5.56% |
| Croston | -4.95% |  | 0.32% | 0.34% |
| SBA | -5.25% | -0.32% |  | 0.02% |
| SBJ | -5.27% | -0.34% | -0.02% |  |
| MAE variation | | | | |
| SES |  | 5.18% | 5.82% | 5.92% |
| Croston | -4.93% |  | 0.61% | 0.70% |
| SBA | -5.50% | -0.61% |  | 0.09% |
| SBJ | -5.59% | -0.69% | -0.09% |  |

Table Errors of item BIP005887 forecast with α = 0.15

As can be seen, once again the Exponential Smoothing shows larger errors, both in RMSE and MAE, compared to the Croston methods, even when the variation of the former (around 5.5%) is smaller than that of the latter (almost 6%). This continues to demonstrate the inaccuracy of this method using the values recommended by the aforementioned authors. Similarly, it can be seen that the Croston method has a slight increase in its errors, when compared to its two modifications (around 0.3%), while SBA and SBJ tend, again, to have quite similar results, but the second method is slightly more accurate, being reduced by 0.2% in its RMSE and 0.09% in its MAE.

As for a greater increase in alpha, in this case to 0.5, it can be seen that, as occurred in the previous analyses, the forecast curves tend to approach the original data, as can be seen in Figure 30. Similarly, both Exponential Smoothing and SBJ work in an indirect proportional way, since while the first tries to increase to be similar to the original values, the second shows the lowest values, but maintaining the tendency to resemble these data. However, the fact that the Exponential Smoothing tries to match the original data in this iteration is still not enough to generate an accurate forecast, as shown in Table 15.

Figure Croston's and Exp. Smooth. forecasts of item BIP005887, α = 0.5

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | SES | Croston | SBA | SBJ |
| ME | 0.24 | -0.23 | 1.21 | 1.69 |
| MAE | 8.03 | 7.66 | 6.93 | 6.70 |
| RMSE | 13.81 | 13.26 | 12.90 | 12.84 |
| Error size | 5.22 | 5.24 | 5.91 | 7.11 |
| RMSE variation | | | | |
| SES |  | 4.20% | 7.07% | 7.60% |
| Croston | -4.03% |  | 2.75% | 3.27% |
| SBA | -6.60% | -2.68% |  | 0.50% |
| SBJ | -7.07% | -3.17% | -0.50% |  |
| MAE variation | | | | |
| SES |  | 4.86% | 15.94% | 19.91% |
| Croston | -4.63% |  | 10.57% | 14.36% |
| SBA | -13.75% | -9.56% |  | 3.43% |
| SBJ | -16.61% | -12.56% | -3.32% |  |

Table Errors of item BIP005887 forecast with α = 0.5

As can be seen, the Exponential Smoothing shows a significant increase in its mean errors, since both its RMSE and its MAE show a lack of precision when applied to items with lumpy demand, which indicates that getting so close to the original values is not synonymous with an adequate forecast. On the other hand, the Croston methods continue with the behavior that was evidenced in the previous iterations, i.e., the original method presents the highest errors and is followed by the SBA and then the SBJ, although between these two the difference, leaving the latter as the most accurate, which has shown that, for lumpy demand items, it seems to be a more recommendable method.

The last iteration performed, with optimal alphas, shows a much more subdued smoothing for all methods in Figure 31, while the behavior of the Croston methods behave similarly. It can even be seen that the Exponential Smoothing, contrary to what happened with alpha = 0.5, shows the lowest trend. Similarly, to the previous types of demand, in this case the optimal alphas for each method are different, as can be seen in Table 16.

Figure Croston's and Exp. Smooth. forecasts of item BIP005887, α = “opt.”

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | SES | Croston | SBA | SBJ |
| α | 0.0615 | 0.07433714 | 0.06683556 | 0.06035939 |
| ME | 1.90 | 0.32 | 0.21 | 0.20 |
| MAE | 6.65 | 7.22 | 7.22 | 7.21 |
| RMSE | 12.72 | 12.59 | 12.50 | 12.48 |
| Error size | 6.07 | 5.36 | 5.28 | 5.26 |
| RMSE variation | | | | |
| SES (0.1) | -0.33% | 4.60% | 4.85% | 4.86% |
| SES (0.15) | -1.15% | 4.00% | 4.33% | 4.35% |
| Croston (0.1) | -1.40% | 3.48% | 3.72% | 3.73% |
| Croston (0.15) | -2.22% | 2.88% | 3.20% | 3.22% |
| SBA (0.1) | -2.05% | 2.80% | 3.04% | 3.05% |
| SBA (0.15) | -2.86% | 2.20% | 2.52% | 2.55% |
| SBJ (0.1) | -2.25% | 2.59% | 2.83% | 2.83% |
| SBJ (0.15) | -3.06% | 1.99% | 2.31% | 2.33% |
| MAE variation | | | | |
| SES (0.1) | -6.75% | -5.79% | -4.99% | -4.98% |
| SES (0.15) | -10.60% | -5.97% | -5.40% | -5.31% |
| Croston (0.1) | 1.23% | 2.28% | 3.15% | 3.16% |
| Croston (0.15) | -2.95% | 2.08% | 2.70% | 2.79% |
| SBA (0.1) | 1.24% | 2.29% | 3.16% | 3.17% |
| SBA (0.15) | -2.93% | 2.09% | 2.72% | 2.81% |
| SBJ (0.1) | 1.11% | 2.16% | 3.03% | 3.04% |
| SBJ (0.15) | -3.06% | 1.96% | 2.59% | 2.68% |

Table Errors of item BIP005887 forecast with α = “optimal”

As can be seen, the alphas used in this optimization do not even exceed the 0.1 limit taken as the lowest value. It is even lower than that of the optimization of the erratic demand items seen in Table 6. However, it is interesting to note that Exponential Smoothing offers a much more accurate forecast than the other methods. Even when comparing its MAE, a great improvement in its accuracy can be observed when compared to the 0.1 and 0.15 iterations. Similarly, the optimization performed for Croston's methods is contradictory, because its MAE and RMSE values are higher than those of the other iterations, so that for these methods, it is more advisable to use an alpha between 0.1 and 0.15 as recommended by the authors analyzed, while for Exponetial Smoothing, it is recommended to use a much lower alpha, which, in practice is not very evident, since in most cases and industries, this value is usually between 0.2 and 0.5.

Finally, when analyzing the forecast of inter-demand intervals in the table 17, the same conclusion can be reached as in the previous iteration: it is not a good tool to help to know when the next demand might occur. This is because it counts the number of periods, backwards, of how many periods there were whose demand was equal to 0, but not a probability of when this demand might or might not occur. As mentioned above, it is possible to identify the times when there is no demand, since this is when the forecast tends to be constant, but in the case that this indicator is used as a tool to predict the next demand, the case could occur in which it is always constant, predicting that there will never be demand, when it does not take into account a probability that indicates whether this is possible or not.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0.1 | | | 0.15 | | | 0.5 | | | opt. | | |
|  | Cros. | SBA | SBJ | Cros. | SBA | SBJ | Cros. | SBA | SBJ | Cros. | SBA | SBJ |
| 17-Dec |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-Jan |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-feb |  |  |  |  |  |  |  |  |  |  |  |  |
| 18-mar | 6.53 | 7.41 | 7.39 | 8.00 | 8.00 | 8.00 | 8.00 | 8.00 | 8.00 | 1.00 | 1.04 | 1.00 |
| 18-Apr | 6.53 | 7.41 | 7.39 | 8.00 | 8.00 | 8.00 | 8.00 | 8.00 | 8.00 | 1.00 | 1.04 | 1.00 |
| 18-may | 6.08 | 6.87 | 6.85 | 7.10 | 7.10 | 7.10 | 5.00 | 5.00 | 5.00 | 1.04 | 1.07 | 1.03 |
| 18-jun | 5.57 | 6.28 | 6.26 | 6.18 | 6.18 | 6.18 | 3.00 | 3.00 | 3.00 | 1.04 | 1.07 | 1.03 |
| 18-jul | 5.57 | 6.28 | 6.26 | 6.18 | 6.18 | 6.18 | 3.00 | 3.00 | 3.00 | 1.04 | 1.07 | 1.03 |
| 18-Aug | 5.57 | 6.28 | 6.26 | 6.18 | 6.18 | 6.18 | 3.00 | 3.00 | 3.00 | 1.04 | 1.07 | 1.03 |
| 18-sep | 5.57 | 6.28 | 6.26 | 6.18 | 6.18 | 6.18 | 3.00 | 3.00 | 3.00 | 1.04 | 1.07 | 1.03 |
| 18-oct | 5.57 | 6.28 | 6.26 | 6.18 | 6.18 | 6.18 | 3.00 | 3.00 | 3.00 | 1.04 | 1.07 | 1.03 |
| 18-nov | 5.57 | 6.28 | 6.26 | 6.18 | 6.18 | 6.18 | 3.00 | 3.00 | 3.00 | 1.04 | 1.07 | 1.03 |
| 18-Dec | 5.61 | 6.25 | 6.24 | 6.16 | 6.16 | 6.16 | 4.50 | 4.50 | 4.50 | 1.25 | 1.22 | 1.18 |
| 19-Jan | 5.61 | 6.25 | 6.24 | 6.16 | 6.16 | 6.16 | 4.50 | 4.50 | 4.50 | 1.25 | 1.22 | 1.18 |
| 19-feb | 5.61 | 6.25 | 6.24 | 6.16 | 6.16 | 6.16 | 4.50 | 4.50 | 4.50 | 1.25 | 1.22 | 1.18 |
| 19-mar | 5.61 | 6.25 | 6.24 | 6.16 | 6.16 | 6.16 | 4.50 | 4.50 | 4.50 | 1.25 | 1.22 | 1.18 |
| 19-Apr | 5.45 | 6.03 | 6.01 | 5.83 | 5.83 | 5.83 | 4.25 | 4.25 | 4.25 | 1.36 | 1.31 | 1.26 |
| 19-may | 5.01 | 5.53 | 5.51 | 5.11 | 5.11 | 5.11 | 2.63 | 2.63 | 2.63 | 1.35 | 1.30 | 1.25 |
| 19-jun | 5.01 | 5.53 | 5.51 | 5.11 | 5.11 | 5.11 | 2.63 | 2.63 | 2.63 | 1.35 | 1.30 | 1.25 |
| 19-jul | 4.70 | 5.17 | 5.16 | 4.64 | 4.64 | 4.64 | 2.31 | 2.31 | 2.31 | 1.38 | 1.32 | 1.27 |
| 19-Aug | 4.33 | 4.76 | 4.75 | 4.10 | 4.10 | 4.10 | 1.66 | 1.66 | 1.66 | 1.36 | 1.31 | 1.27 |
| 19-sep | 4.00 | 4.38 | 4.37 | 3.63 | 3.63 | 3.63 | 1.33 | 1.33 | 1.33 | 1.34 | 1.30 | 1.26 |
| 19-oct | 3.70 | 4.04 | 4.03 | 3.24 | 3.24 | 3.24 | 1.16 | 1.16 | 1.16 | 1.33 | 1.29 | 1.25 |
| 19-nov | 3.43 | 3.74 | 3.73 | 2.90 | 2.90 | 2.90 | 1.08 | 1.08 | 1.08 | 1.32 | 1.29 | 1.24 |
| 19-Dec | 3.19 | 3.46 | 3.46 | 2.62 | 2.62 | 2.62 | 1.04 | 1.04 | 1.04 | 1.30 | 1.28 | 1.24 |
| 20-Jan | 3.19 | 3.46 | 3.46 | 2.62 | 2.62 | 2.62 | 1.04 | 1.04 | 1.04 | 1.30 | 1.28 | 1.24 |
| 20-feb | 3.07 | 3.32 | 3.31 | 2.52 | 2.52 | 2.52 | 1.52 | 1.52 | 1.52 | 1.33 | 1.30 | 1.26 |
| 20-mar | 2.86 | 3.09 | 3.08 | 2.30 | 2.30 | 2.30 | 1.26 | 1.26 | 1.26 | 1.32 | 1.29 | 1.25 |
| 20-Apr | 2.86 | 3.09 | 3.08 | 2.30 | 2.30 | 2.30 | 1.26 | 1.26 | 1.26 | 1.32 | 1.29 | 1.25 |
| 20-may | 2.86 | 3.09 | 3.08 | 2.30 | 2.30 | 2.30 | 1.26 | 1.26 | 1.26 | 1.32 | 1.29 | 1.25 |
| 20-jun | 2.86 | 3.09 | 3.08 | 2.30 | 2.30 | 2.30 | 1.26 | 1.26 | 1.26 | 1.32 | 1.29 | 1.25 |
| 20-jul | 2.86 | 3.09 | 3.08 | 2.30 | 2.30 | 2.30 | 1.26 | 1.26 | 1.26 | 1.32 | 1.29 | 1.25 |
| 20-Aug | 2.86 | 3.09 | 3.08 | 2.30 | 2.30 | 2.30 | 1.26 | 1.26 | 1.26 | 1.32 | 1.29 | 1.25 |
| 20-sep | 2.86 | 3.09 | 3.08 | 2.30 | 2.30 | 2.30 | 1.26 | 1.26 | 1.26 | 1.32 | 1.29 | 1.25 |
| 20-oct | 2.86 | 3.09 | 3.08 | 2.30 | 2.30 | 2.30 | 1.26 | 1.26 | 1.26 | 1.32 | 1.29 | 1.25 |
| 20-nov | 3.38 | 3.58 | 3.57 | 3.15 | 3.15 | 3.15 | 4.63 | 4.63 | 4.63 | 1.60 | 1.50 | 1.45 |

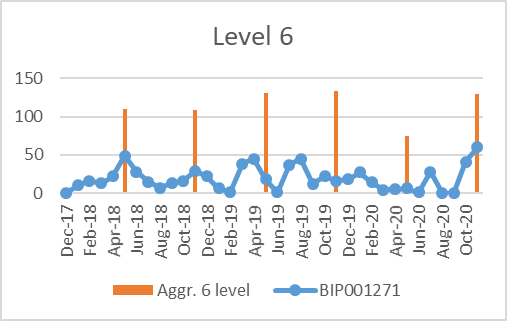
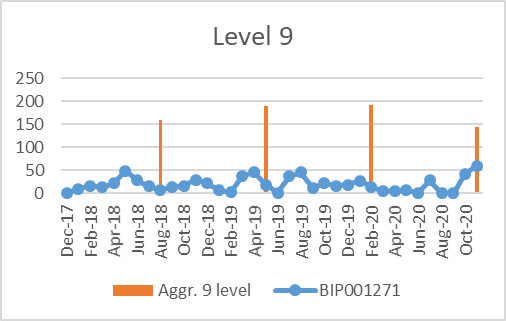
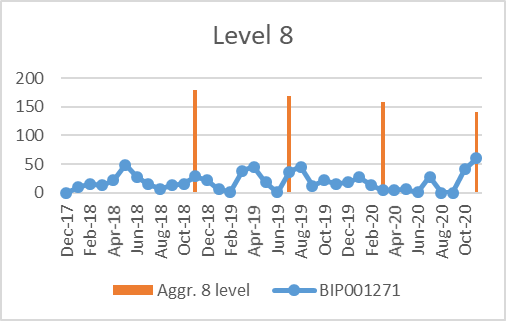
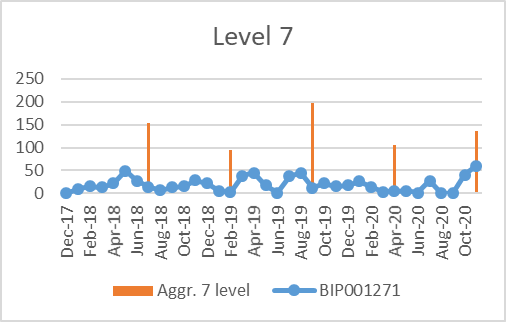
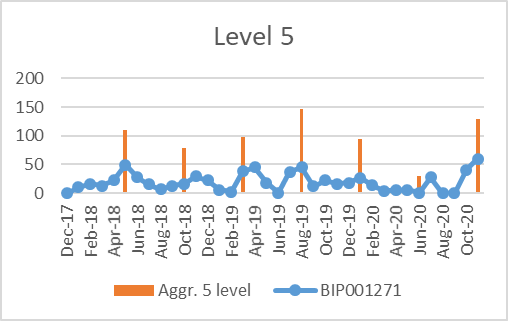
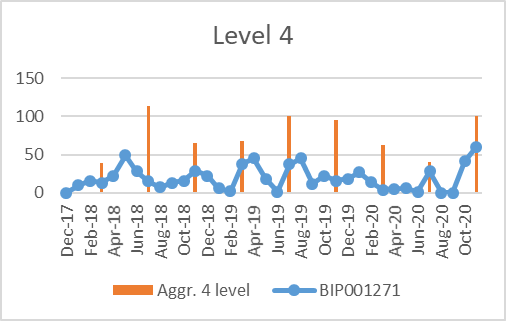
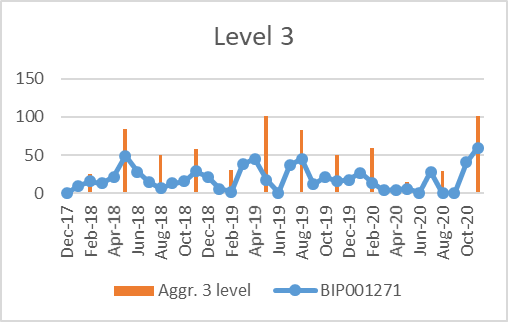
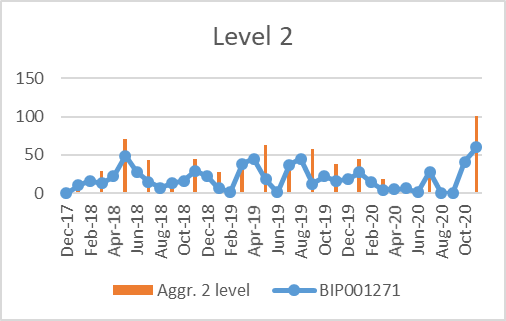
Table 17 Smoothed inter-demand interval of item BIP005887 (intermittent demand)

## 4.2. Aggregate-Disaggregate Intermittent Demand Approach (ADIDA) and Multiple Aggregation Prediction Algorithm (MAPA)

As mentioned in chapter 3, these two forecasting methods aim to "fill" the periods with zero demand by aggregating the time series into one of lower frequency. In the case of ADIDA, this must be entered manually, i.e., a specific aggregation level must be thought of. In contrast, MAPA iterates through all the aggregation levels it could have and generates a forecast using some method for this purpose. Thus, in the case of the first method, aggregation will be observed at two levels (bimonthly) while the other will observe aggregation at up to 9 levels (from monthly to nine monthly). Similarly, since the items analyzed are slow-moving demand items, the methods used by these two procedures are Croston, SBA and Exponential Smoothing, which are specialized for this type of items. Finally, for these methods, at each level of aggregation, the chosen method is justified through the evaluation of the Coefficient of Variation (CV2) and the Average Inter-Demand Interval, in addition to the classification of demand patterns established by Syntetos et al. (2005).

### 4.2.1. Erratic items

For the erratic items, Figure 32 shows their aggregation from level 2 to level 9. In this, it can be seen at a glance that the variation in the size of the demand has decreased, which implies that the Coefficient of Variation has also decreased (from 0.5719 in the original data to 0.3292 in the aggregation for the case of level 2 and to 0.0176 for the case of level 9). In the same way, the Average Inter-Demand Interval fluctuates negatively, decreasing from 1.09 to 1.05 for level 2 and to 1 level 9. Thanks to this, the evaluated item goes gradually from being an erratic type to a smooth type, so that, when forecasting its demand, the Croston method or the Exponential Smoothing can be used, since in this type of items, the two results will tend to the same (Petropoulos & Kourentzes, 2014).



Aggregation levels for Item BIP001271

Figure 32 Aggregation levels for Item BIP001271

Based on these aggregations, and the changes that the time series goes through during this process, the choice of the forecasting method for slow-moving items to which they will be subjected, both in ADIDA and MAPA, will depend on the parameters mentioned above (Coefficient of Variation and Average Inter-Demand Interval - ADI) and the organization set forth by Syntetos & Boylan (2005) on which methods are applied to the demand patterns with better accuracy. Thus, Table 18 shows the values obtained by each level of aggregation with its respective forecasting method. It is important to clarify that, according to Petropoulos & Kourentzes (2014), when the ADI is equal to 1, the use of Exponential Smoothing is recommended, since the number of zeros during the time series disappears, in addition to generating less bias than when applying a method designed for slow-moving items, such as the SBA.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Aggregation level | | | | | | | |
|  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Obs. | 18 | 12 | 9 | 7 | 6 | 5 | 4 | 4 |
| ADI | 1.06 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| CV2 | 0.33 | 0.27 | 0.13 | 0.15 | 0.04 | 0.09 | 0.01 | 0.02 |
| Model | Croston | SES | SES | SES | SES | SES | SES | SES |

Table 18 Selection of forecasting method for ADIDA and MAPA - BIP001271

Subsequently, establishing the methods to be applied at each time level, being practically ADIDA part of MAPA, but without combining them, we proceed to the execution of the forecast. Thus, Figure 33 shows the curves generated by executing these methods. It can be seen that MAPA shows a much more constant result than its second level of aggregation, i.e. ADIDA, which could be evidence of a better accuracy in its final forecast. Table 19 shows the smoothed results for each period, as well as for the following 5 periods as established in Croston's methods. The complete data will be shown in the annex.

Figure 33 Aggregation-Disaggregation forecasting methods for item BIP001271

|  |  |  |  |
| --- | --- | --- | --- |
| X | BIP001271 | ADIDA | MAPA |
| Dec-17 | 0 |  |  |
| Jan-18 | 10 |  |  |
| Feb-18 | 16 | 6.15 |  |
| Mar-18 | 13 | 6.15 |  |
| Apr-18 | 22 | 8.41 | 17.76 |
| May-18 | 49 | 8.41 | 17.80 |
| Jun-18 | 28 | 15.76 | 18.91 |
| Jul-18 | 15 | 15.76 | 18.98 |
| Aug-18 | 7 | 17.31 | 19.13 |
| Sep-18 | 13 | 17.31 | 19.01 |
| Oct-18 | 16 | 15.33 | 18.73 |
| Nov-18 | 29 | 15.33 | 18.70 |
| Dec-18 | 22 | 17.28 | 19.05 |
| Jan-19 | 6 | 17.28 | 19.07 |
| Feb-19 | 2 | 16.39 | 18.85 |
| Mar-19 | 38 | 16.39 | 18.68 |
| Apr-19 | 45 | 17.37 | 18.97 |
| May-19 | 18 | 17.37 | 19.20 |
| Jun-19 | 1 | 21.20 | 19.63 |
| … | … | … | … |
| Jul-20 | 28 | 14.92 | 18.21 |
| Aug-20 | 0 | 14.81 | 18.27 |
| Sep-20 | 0 | 14.81 | 18.25 |
| Oct-20 | 41 | 14.81 | 18.25 |
| Nov-20 | 60 | 14.81 | 18.46 |
| Forecast 1 |  | 12.24 | 18.39 |
| Forecast 2 |  | 12.24 | 18.39 |
| Forecast 3 |  | 12.24 | 18.39 |
| Forecast 4 |  | 12.24 | 18.39 |
| Forecast 5 |  | 12.24 | 18.39 |

Table 19 Aggregation – Disaggregation smoothing and forecast of item BIP001271

On the other hand, when evaluating the errors of this iteration, Table 20 shows that when using time series aggregation at only one level, as in the case of ADIDA, there is a relatively wide margin in which the forecast is more inaccurate than when combining several levels of aggregation, as in the case of MAPA. In both its RMSE and MAE, the latter method shows a higher accuracy, hence a more feasible application than the former (about 5% and 2% respectively). This justifies that, with the combination of the forecast components of each of the aggregation levels in MAPA, the benefits of each of them can be obtained to approximate a less biased forecast with higher accuracy. However, whether this approach is better than traditional methods such as Croston and Exponential Smoothing will be reviewed later.

|  |  |  |
| --- | --- | --- |
|  | ADIDA | MAPA |
| ME | 3.21 | 2.18 |
| MAE | 13.01 | 12.74 |
| RMSE | 16.49 | 15.65 |
| Error size | 3.48 | 2.91 |
| RMSE variation | | |
| ADIDA |  | 5.38% |
| MAPA | -5.11% |  |
| MAE variation | | |
| ADIDA |  | 2.07% |
| MAPA | -2.03% |  |

Table 20 Aggregation errors for item BIP001271

### 4.2.2. Intermittent items

Since all items used in this study have the same time limit, i.e. from December 2017 to October 2020, the aggregation levels will be the same. Thus, Figure 34 shows the 9 aggregation levels to which the evaluated item was subjected. It can be observed that, given its intermittency, the first levels will not have such a stable aggregation, due to the fact that there are periods in which the demand equal to zero is longer, mainly with period 2, that is, with the ADIDA iteration that this study will have. However, by checking with Table 21, interesting differences can be seen with respect to the erratic items.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Aggregation level | | | | | | | | |
|  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Obs. | 18 | 12 | 9 | 7 | 6 | 5 | 4 | 4 |
| ADI | 1.64 | 1.33 | 1.29 | 1.40 | 1.20 | 1.25 | 1.00 | 1.33 |
| CV | 0.60 | 0.56 | 0.67 | 0.41 | 0.71 | 0.39 | 0.79 | 0.19 |
| Model | SBA | SBA | SBA | SBA | SBA | SBA | SES | SBA |

Table Selection of forecasting method for ADIDA and MAPA - BIP008013

As can be seen, due to the large number of zeros along the time series, it is possible to see the transformation of the demand behavior from an intermittent type item to a lumpy type item and back to an intermittent type item. Even at the fourth level, the variation of its demand increases to the point of becoming an erratic type item. Thus, thanks to these transformations, it can be seen that the predominant method to be established in each of them will be the SBA, except for level 8, due to its ADI = 1, which, as explained above, generates better results when subject to Exponential Smoothing. In this way, when individually performing the application of the methods and finding the components of each one of them, they are combined, using the mean. It should be noted that this process is only applied for MAPA, since for ADIDA, having the smoothed values of the aggregation, it should only be disaggregated at the initial temporal level. Figure 35 shows these two procedures.

Aggregation levels for Item BIP008013

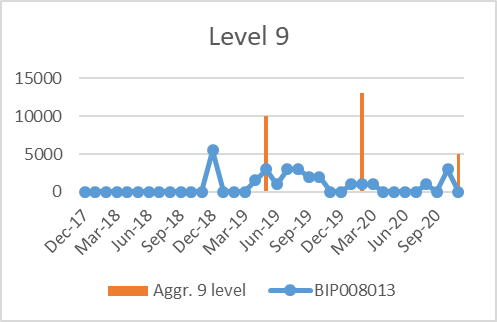
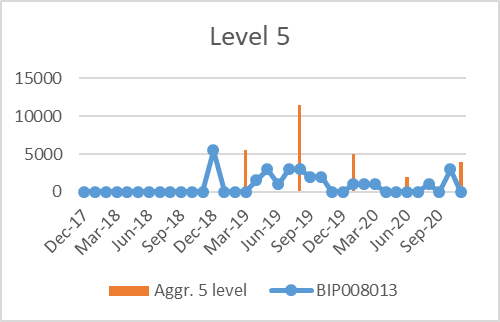
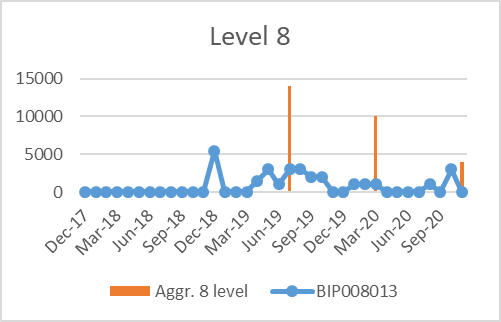
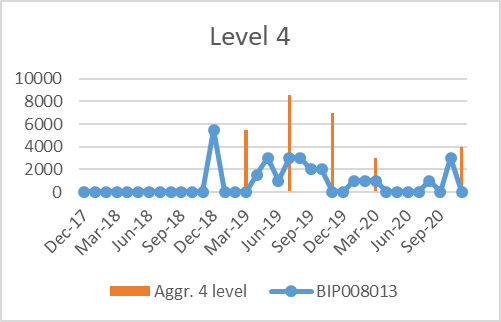
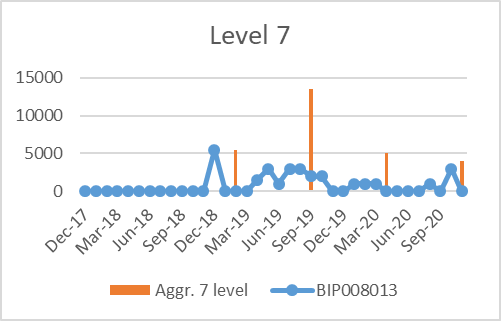
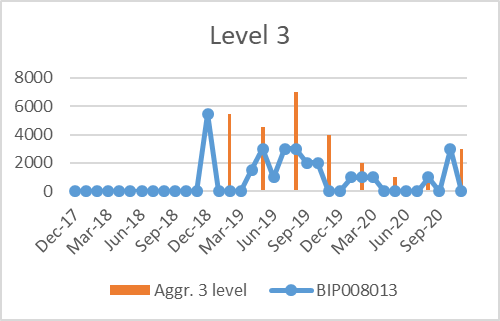
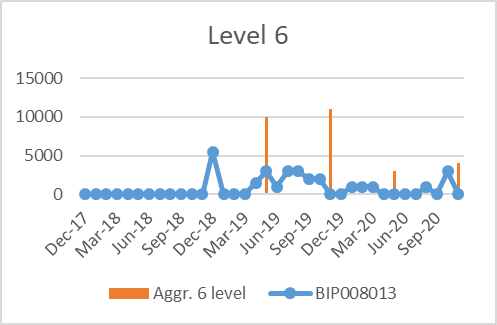
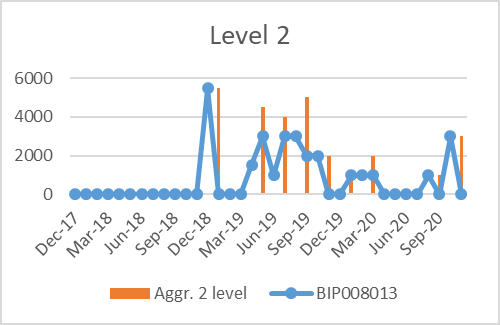


Figure 34 Aggregation levels for Item BIP008013

Figure 35 Aggregation-Disaggregation forecasting methods for item BIP008013

Based on this and the review of the errors found in Table 22, it can still be observed that MAPA offers a higher accuracy than its counterpart ADIDA. With a negative difference of 1.58% in its MAE, the combination of several aggregation levels is a much more accurate tool than choosing a random aggregation level or depending on the lead time. Likewise, the smoothing benefits generated by applying the methods at the aggregation levels and combining them can be perceived. However, it is interesting to note that the RMSE is higher in MAPA than in ADIDA, although as mentioned in the Croston methods, this indicator may not be very adequate to evaluate the accuracy of these items given the high number of periods with zero demand.

|  |  |  |
| --- | --- | --- |
|  | ADIDA | MAPA |
| ME | 340.88 | 376.36 |
| MAE | 752.58 | 740.70 |
| RMSE | 1330.59 | 1347.18 |
| Error size | 578.01 | 606.48 |
| RMSE variation | | |
| ADIDA |  | -1.23% |
| MAPA | 1.25% |  |
| MAE variation | | |
| ADIDA |  | 1.60% |
| MAPA | -1.58% |  |

Table 22 Table 20 Aggregation errors for item BIP008013

### 4.2.3. Lumpy demand

The last type of items to be analyzed with these forecasting methods are those with lumpy demand. Figure 36 shows the behavior of the aggregation levels to which the analyzed item was exposed, where it can be seen that aggregation level 2 is still insufficient to "eliminate", so to speak, the periods with demand equal to zero. In any case, as observed in the previous cases, as the aggregations increase, the type of item is transformed into other types. This can be seen in Table 23, where it is interesting to see the transformation of this series from lumpy to erratic, without becoming intermittent. Similarly, it can also be observed that the type of model that would be applied to each level of aggregation is mainly between SBA and Exponential Smoothing, given the approximation of the ADI to 1.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Aggregation level | | | | | | | | |
|  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Obs. | 18 | 12 | 9 | 7 | 6 | 5 | 4 | 4 |
| ADI | 1.64 | 1.50 | 1.13 | 1.17 | 1.00 | 1.00 | 1.00 | 1.00 |
| CV | 1.27 | 1.10 | 0.98 | 0.89 | 0.87 | 0.88 | 0.31 | 0.55 |
| Model | SBA | SBA | SBA | SBA | SES | SES | SES | SES |

Table 23 Selection of forecasting method for ADIDA and MAPA - BIP005887

On the other hand, when reviewing the application of the two methods, it can be seen in Figure 37 that ADIDA tends to be more affected by demand peaks than MAPA, due to the combination of the components of the other temporal aggregations. It is also interesting to note that regardless of these demand peaks, the final forecast tends to be very similar, so that possibly the size of the demand affects the two methods, and their results do not vary too much, given that the size of the demand for this item is much smaller than in the previous cases.

Aggregation levels for Item BIP005887

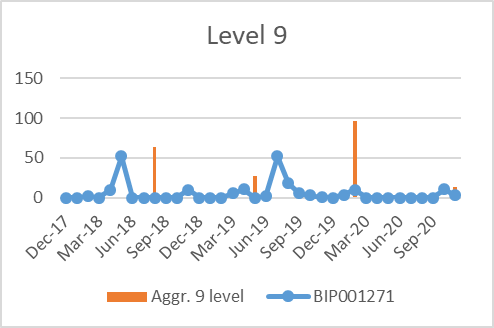
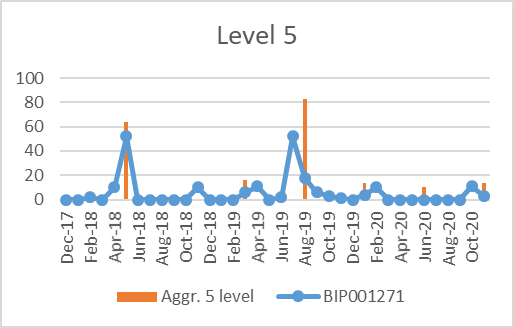
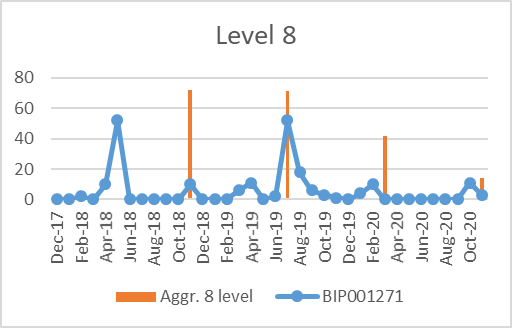
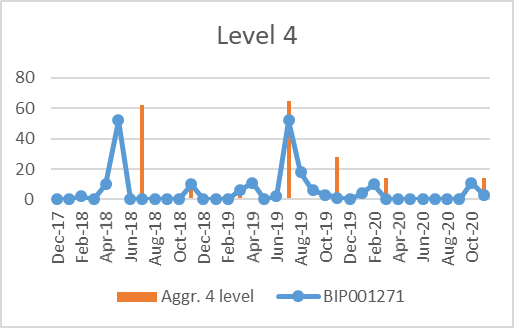
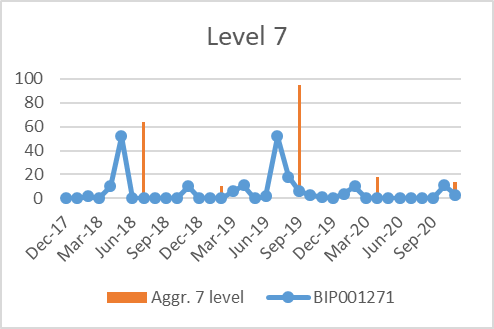
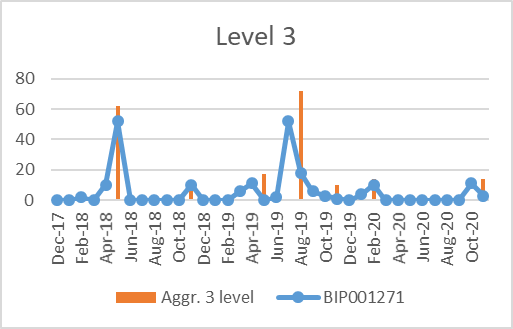
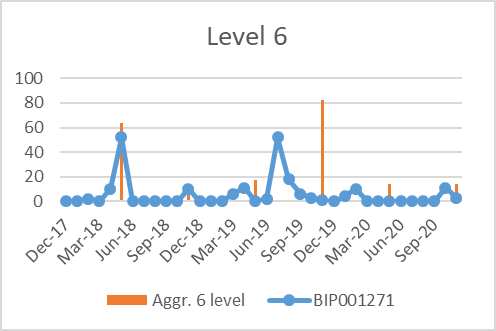
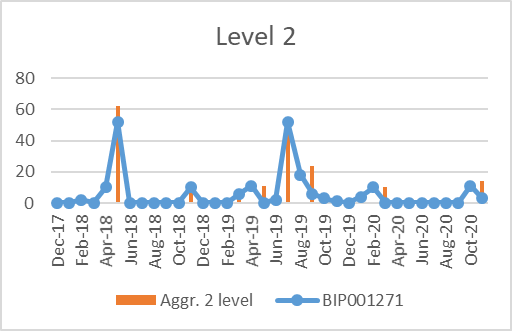


Figure 36 Aggregation levels for Item BIP005887

Figure 37 Aggregation-Disaggregation forecasting methods for item BIP005887

Finally, Table 24 shows the forecast errors for these types of items. It is surprising to note that ADIDA, in this iteration, shows more stable and therefore more accurate results than MAPA. Similarly, it can be observed that the two error measurement indicators used agree with this assertion, although in the MAE the differences are not so large (ADIDA's negative variation of 0.19%), possibly due to the small size of the demand for the item analyzed. On the other hand, the RMSE of ADIDA, although it has a negative variance of 0.65% with respect to MAPA, is still very small, which implies that both methods tend to have similar precision, although the former is more accurate than the latter. Similarly, it is important to review the performance of the other methods evaluated above and contrast them, so that the best forecasting method under these parameters can be established.

|  |  |  |
| --- | --- | --- |
|  | ADIDA | MAPA |
| ME | 0.51 | 0.57 |
| MAE | 7.26 | 7.27 |
| RMSE | 12.72 | 12.80 |
| Error size | 5.46 | 5.53 |
| RMSE variation | | |
| ADIDA |  | -0.65% |
| MAPA | 0.66% |  |
| MAE variation | | |
| ADIDA |  | -0.19% |
| MAPA | 0.19% |  |

Table Table 22 Table 20 Aggregation errors for item BIP005887

## 4.3. Final comparison between the aforementioned methods

Given the final results and thanks to the review of the errors of each of the applied methods, several interesting things can be evidenced when comparing and contrasting the two types of implemented models. Thus, for each type of item, an analysis of the comparison of their methods will be provided and the one that showed the best performance, according to the errors evidenced.

### 4.3.1 Erratic demand items

In the case of the erratic items, in the 3 basic scenarios (alpha = 0.1, 0.15 and 0.5) the Exponential Smoothing did not show good precision, neither in the MAE nor in the RMSE. This indicates that in this method the less recent values are not taken with such a high priority when forecasting. Similarly, given the stability of the time series, it is understandable why the obtained value of alpha is 0.32, since it tends to have a better result when more recent values are taken into account. Even when the value of this variable is equal to 0.5, the forecast results are much more accurate than with small alphas. Within this framework, the Exponential Smoothing has behaved as several authors have stated.

On the other hand, when reviewing Croston's methods, they show a very similar precision between them during all the alphas evaluated. It is also noteworthy that these methods, despite being based on Exponential Smoothing, showed a better performance than this method. Similarly, among them, an evident improvement can be observed between the SBA and the SBJ with respect to the pioneer model (Croston), which indicates that the corrections proposed by Syntetos & Boylan (2005) and Shale et al. (2006) improve the accuracy of the forecasts. However, it is noticed that these methods tend to forecast demands with the same size, i.e., the values of their RMSE and MAE tend to be very close to each other, so, practically, using one method or the other would give similar results.

However, when comparing the Exponential Smoothing, those based on Croston and those on Aggregation, we can observe that, surprisingly, SBA-SBJ (in this case together due to their closeness in their results), present the best conditions to forecast items with erratic demand, using an alpha between 0.1 and 0.15. Despite the combinations made by the aggregation models and the fact that, exclusively, the MAE uses and combines some of the methods mentioned above, it is not enough to generate a more adequate forecast than that of SBA-SBJ. Despite this, it is important to emphasize that, since the alphas are small, priority is being given to the initial periods of the time series when making the forecast, which in the case analyzed in this study applies correctly, since in these periods there is some stability in the size and periodicity of demand. With all this said, it is emphasized that the best methods for erratic items are SBA-SBJ, which also contradicts what some authors have established about the application of this methods in this kind of items (Kaya et al., 2020).

### 4.3.2. Intermittent demand items

Regarding intermittent demand items, the results of the Exponential Smoothing type methods (including those of Croston) showed heterogeneous results, with respect to the previous classification. This is demonstrated by the fact that during the iterations carried out, some methods were better than others, but their precision changed when using another alpha. In this way, when alpha is 0.1, SBA-SBJ were the most suitable, both in its MAE and in its RMSE (it should be noted that, from this method, the RMSE is not as accurate as a tool to measure the precision of these methods, due to the irregularity of the time series, as well as the weight that affects the use of high value errors), with alpha equal to 0.15 the best was the Exponential Smoothing, with alpha equal to 0.5 SBJ showed the best precision and with alpha optimal, the best was the Exponential Smoothing with an alpha of 0.18, keeping the trend that it had in the second case explained above. This indicates that the application of these methods is relative and circumstantial, i.e., depending on how the data are located in the time series, it could generate one or another result, because when observing the optimal values, these are not so small. as in the case of erratic items, due to the lack of initial demand in the time series (so the oldest values are not being given such a high weight). This is interesting, in the sense that most authors state that, in the case of intermittent demand items, the method to use should be SBA.

On the other hand, when observing the aggregation methods, an improvement can be seen when grouping the data at various levels of aggregation, since, ultimately, MAPA generates the best results. It is observed that in their MAE, both ADIDA and the previous one have competent results with the Exponential Smoothing of the optimal phase. However, of these it is better to select MAPA, since its MAE is much lower than that of the other methods, in addition to taking into account the benefits of the components resulting from the forecast of each of the levels of aggregation to which it was exposed. the time series. With all this, the best method to forecast items with intermittent demand is the MAPA.

### 4.3.3. Lumpy demand items

Finally, when analyzing the results of the items with lumpy-type demand, a moderately similar behavior to that of the erratic items can be evidenced, in the sense of the heterogeneity of the results. When using small alphas, much less than 1.0 (mainly evidenced in the optimization of this parameter), the best method to forecast is the Exponential Smoothing, even going against what Syntetos & Boylan (2001) evidenced, about even the Croston method would behave better with alphas less than 1.5. However, when increasing alpha, it was observed that the Exponential Smoothing was losing more precision, while the other methods improved it, so this method attributes more weight to initial data than the Croston type, which consider average or more current values. to improve your forecasts. Thus, for alphas higher than to 0.1, the best scheme was SBA-SBJ.

With respect to the aggregation methods, their development was not as expected, since neither ADIDA nor MAPA will show inferior results to those previously analyzed. This phenomenon is interesting, since they should have generated much more competitive details given the methodology they use. However, given the results obtained, these methods could not be taken into account to make more accurate forecasts.

Based on the above, two types of scenarios could be generated for the use of the methods: the first, if the less recent data wants to have a greater influence on the forecast, you can opt for the use of the Exponential Smoothing with a lower value to 0.1, while if the most recent data are those to which you want to give a greater preponderance, the option is SBA-SBJ, with greater use of SBJ if the alpha is from 0.5 onwards.

# REFERENCES

Billah, B., King, M. L., Snyder, R. D., & Koehler, A. B. (2006). Exponential smoothing model selection for forecasting. *International Journal of Forecasting, 22*(2), 239-247.

Boylan, J. E., & Syntetos, A. A. (2007). The accuracy of a Modified Croston procedure. *International Journal of Production Economics, 107*(2), 511-517.

Boylan, J. E., Syntetos, A. A., & Karakostas, G. C. (2008). Classification for forecasting and stock control: a case study. *Journal of the Operational Research Society, 59*, 473-481.

Costantino, F., Di Gravio, G., Patriarca, R., & Petrella, L. (2018). Spare parts management for irregular demand items. *Omega, 81*, 57-66.

Croston, J. D. (1972). Forecasting and Stock Control for Intermittent Demands. *Operational Research Quarterly, 23*(3), 289-303.

Dunsmuir, W. T., & Snyder, R. N. (1989). Control of inventories with intermittent demand. *European Journal of Operational Research, 40*(1), 16-21.

Gardner, E. S. (1985). Exponential Smoothing: The State of art. *Journal of Forcasting, 4*(1), 1-28.

Johnston, F. R., & Boylan, J. E. (1996). Forecasting intermittent demand: a comparative evaluation of Croston's method. Comment. *International Journal of Forecasting, 12*(2), 297-298.

Johnston, F. R., Boylan, J. E., & Shale, E. A. (2003). An examination of the size of orders from customers, their characterisation and the implications for inventory control of slow moving items. *Journal of the Operational Research Society, 54*(8), 833-837.

Kaya, G. O., Sahin, M., & Demirel, O. F. (2020). Intermittent demand forecasting: a guideline for method selection. *Sadhana, 45*, 51.

Kourentzes, N., & Petropoulos, F. (2016). Forecasting with R. *International Symposium on Forecasting 2016* (p. 65). Santander: International Institute of Forecasters.

Kourentzes, N., Petropoulos, F., & Trapero, J. R. (2014). Improving forecasting by estimating time series structural components across multiple frequencies. *International Journal of Forecasting, 30*, 291-302.

Leven, E., & Segerstedt, A. (2004). Inventory control with a modified Croston procedure and Erlang distribution. *International Journal of Production Economics, 90*, 361-367.

Montgomery, D. C., Jennings, C. L., & Kulahci, M. (2016). *Introduction to Time Series Analysis and Forecasting.* Hoboken, New Jersey: John Wiley & Sons, Inc.

Nikolopoulos, K. (2021). We need to talk about intermittent demand forecasting. *European Journal of Operational Research, 291*(2), 549-559.

Nikolopoulos, K., Syntetos, A. A., Boylan, J. E., Petropoulos, F., & Assimakopoulos, V. (2011). An Aggregate-Disaggregate Intermittent Demand Approach (ADIDA) to forecasting: An empirical proposition and analysis. *Journal of the Operational Research Society, 62*(3), 544-554.

Petropoulos, F., & Kourentzes, N. (2014). Forecast combinations for intermittent demand. *Journal of the Operational Research Society, 66*(6), 914-924.

Petropoulos, F., Kourentzes, N., & Nikolopoulos, K. (2016). Another look at estimators for intermittent demand. *International Journal of Production Economics*, 154-161.

Ryu, K., & Sanchez, A. (2003). The Evaluation of Forecasting Methods at an Institutional Foodservice Dining Facility. *The Journal of Hospitality Financial Management, 11*(1), 27-45.

Sahin, M., Kizilaslan, R., & Demirel, O. F. (2013). Forecasting Aviation Spare Parts Demand Using Croston Based Methods and Artificial Neural Networks. *Journal of Economic and Social Research, 15*(2), 1-21.

Santa Cruz, R., & Correa, C. (2017). Intermittent demand forecasting with time series methods and artificial neural networks: A case study. *Dyna, 84*(203), 9-16.

Segerstedt, A., & Levén, E. (2020). *A study of different Croston-like forecasting methods.* Lulea, Sweden: Lulea University of Technology.

Shale, E. A., Boylan, J., & Johnston, F. R. (2006). Forecasting for intermittent demand: the estimation of an unbiased average. *Journal of the Operational Research Society, 57*(5), 588-592.

Silver, E. A. (1981). Operations Research in Inventory Management: A Review and Critique. *Operations Research, 29*(4), 628-645.

Silver, E. A., Pyke, D. F., & Peterson, R. (1998). *Inventory Management and Production Planning and Scheduling.* New York, United States of America: John Wiley & Sons, Inc.

Spithourakis, G. P., Petropoulos, F., Nikolopoulos, K., & Assimakopoulos, V. (2014). A systemic view of the ADIDA framework. *IMA Journal of Management Mathematics, 25*(2), 125-137.

Stevenson, W. J. (2009). *Operations Management.* New York: McGraw-Hill/Irwin.

Syntetos, A. A., & Boylan, J. E. (2001). On the bias of intermittent demand estimates. *Intenational Journal of Production Economics, 71*, 457-466.

Syntetos, A. A., & Boylan, J. E. (2005). The accuracy of intermittent demand estimates. *International Journal of Forecasting, 21*, 303-314.

Syntetos, A. A., Boylan, J. E., & Croston, J. D. (2005). On the categorization of demand patterns. *Journal of the Operational Research Society, 56*, 495-503.

Teunter, R. H., & Duncan, L. (2009). Forecasting intermittent demand: a comparative study. *Journal of the Operational Research Society*, 321-329.

Teunter, R. H., Syntetos, A. A., & Zied Babai, M. (2011). Intermittent demand: Linking forecasting to inventory obsolescence. *European Journal of Operational Research, 214*, 606-615.

Tliche, Y., Taghipour, A., & Canel-Depitre, B. (2020). An improved forecasting approach to reduce inventory levels in decentralized supply chains. *European Journal of Operational Research, 287*(2), 511-527.

Uzunoglu Kocer, U., & Tamer, S. (2011). Determining the Inventory Policy for Slow-Moving Items: A Case Study. *Proceedings of the World Congress on Engineering* (pp. 1-4). London: International Association of Engineers.

van Steenbergen, R. M., & Mes, M. R. (2020). Forecasting demand profiles of new products. *Decision Support Systems, 139*, 113401.

Vinh, D. Q. (2005). Forecasting irregular demand for spare parts inventory. *Department of Industrial Engineering, Pusan National University, Busan*, 609-735.

Wallstrom, P., & Segerstedt, A. (2010). Evaluation of forecasting error measurements and techniques for intermittent demand. *International Journal of Production Economics, 128*, 625-636.

Willeman, T. R., Smart, C. N., Shockor, J. H., & DeSautels, P. A. (1994). Forecasting intermittent demand in manufacturing: A comparative evaluation of Croston’s method. *International Journal of Forecasting, 10*, 529-538.

Williams, T. M. (1984). Stock Control with Sporadic and Slow-Moving Demand. *Journal of the Operational Research Society, 35*(10), 939-948.

Xu, Q. Z., Wang, N., & Shi, H. P. (2012). A Review of Croston’s Method for Intermittent Demand Forecasting. *9th International Conference on Fuzzy Systems and Knowledge Discovery* (pp. 1468-1472). Chongqing: IEEE.